

THE BENACERRAF'S DILEMMA

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My intention in this short paper is to discuss some of the problems (along with selected responses) raised by an article *Mathematical Truth* published in the *Journal of Philosophy* in 1973. The essay, penned by Paul Benacerraf, was prepared for and presented on a symposium devoted to the topic implied by the title: mathematical truth. The problem for which it is popular, called “the Benacerraf’s dilemma” (or alternatively “the Access Problem”, “the Reliability Challenge”, and the “Benacerraf-Field Challenge”), has shaped and dominated the discourse in philosophy of mathematics since the moment of the publication in 1973 and continues to do so even to this day. Even though *Mathematical Truth* did not present any novel solutions to existing problems in the field, it has made an impact by shedding new light on questions like: how can we have access to seemingly abstract mathematical objects? What is the nature of mathematical knowledge and do we have any? Is mathematical language part of natural language or is it something quite different? PB has successfully shown the connection between concepts of truth, knowledge, semantics and syntax in mathematics, thus providing a new perspective on thinking about those questions.

The dilemma in short is that the conditions required for mathematical truth make mathematical knowledge impossible and, in reverse, covering epistemology in a way that enables us to achieve mathematical knowledge causes problems in accounting for truth in mathematics. At the beginning of the paper Benacerraf contends that accounts of mathematical truth were driven by two distinct concerns:

- (1) the concern for having a homogenous semantical theory in which semantics for the propositions of mathematics parallel the semantics for the rest of the language.
- (2) The concern that the account of mathematical truth meshes with a reasonable epistemology¹

Benacerraf’s proceeds to postulate that those concerns have always been pursued at the expense of the another and never jointly:

¹Paul Benacerraf, ‘Mathematical Truth’, *The Journal of Philosophy* LXX, 1973, p.661

Accounts of truth that treat mathematical and non-mathematical discourse in relevantly similar ways do so at the cost of leaving it unintelligible how we can have any mathematical knowledge whatsoever; whereas those which attribute to mathematical propositions the kinds of truth conditions we can clearly know to obtain, do so at the expense of failing to connect these conditions with any analysis of the sentences which shows how the assigned conditions are conditions of their *truth*.²

Since a brief exploration of the two opposing tendencies in philosophy has revealed them to be at cross-purposes, two horns of the dilemma can be intelligibly distinguished in a following way: 1) semantical and is a sense metaphysical since semantics indirectly lead us to ontological status of mathematical objects as we will hopefully see, 2) epistemological, i.e. dealing with knowability of mathematical truths.

Benacerraf draws a distinction along the same lines: there are two major requirements (or constraints) for over-all view of truth in mathematics that need to be fulfilled to be deemed satisfactory and philosophically viable. First requirement demands for mathematical truth that the account fits into a theory of truth that covers language as a whole. Benacerraf stipulates that this requirement might amount to the plea that 'the semantical apparatus of mathematics be seen as a part and parcel of that of the natural language'³, however the most important point might be a negative one: the conditions for truth of mathematical propositions, he argues, must clearly indicate that fulfilling them secures that what we are dealing is indeed truth, not *simply* crude theoremhood in a formal system. Although *prima facie* it seems to be a straightforward dismissal of formalist approach, it can be redeemed if one clarifies "the connection between truth and theoremhood".⁴ This kind of account, however, had not been yet presented, adds Benacerraf. He explicitly states his sympathy for to the view that a successful account was provided by Tarski and wants mathematical truth to be defined in terms of reference.⁵

The epistemological constraint consists in the supposition that our over-all views must be such that the account of mathematical truth is incompatible with rejection of mathematical knowledge as non-existent. In other words, there must be *some* mathematical truths knowable to

²Ibid., p.662

³Ibid. p.666

⁴Ibid.

⁵Ibid.

us. This, on itself, does not imply that all mathematical truths are knowable or that only mind can be repository of mathematical truths. As Steven Shapiro puts it “there may be objective truths about mind-dependent entities”⁶ and Benacerraf seems to think along the same lines. Furthermore, the fact *that* we have mathematical knowledge must be intelligibly explained in a manner that would fit more general epistemology.⁷ This is to mean that to Benacerraf invoking extranatural faculties does not seem to be a compelling case for explanation of mathematical knowledge.

Those two requirements are at the very core of the Benacerraf's Dilemma, however the philosophical premisses on which they are based were not left unchallenged. On the interpretation of W.D Hart⁸, the metaphysical landscape depicted in this “semantic” requirement presents itself as follows: ‘truth is a matter of relations between the words and the world’.⁹In order for correspondence to constitute truth, a structure comprised of singular terms must parallel what really is “out there”. Hence, reference to objects is necessary condition for truth. Another important point is that on Benacerraf's view, mathematics is a body of truths. While this this general metaphysical position might not be *wrong per se*, it left some philosophers cold. Richard Creath (1980)¹⁰ argued that treating both ‘semantical homogeneity’ and ‘reasonable epistemology’ as if they were equal is neither justified nor particularly efficient. According to Creath, the first horn of the dilemma, concern for having a homogenous semantics for ‘mathamatese’ and natural language is to some extent a matter of esthetic taste or at any rate constitutes significantly less legitimate endeavor than securing mathematical knowledge, which, as we pointed out, has catalyzed the formalist approaches. Moreover, RC brings forth the idea that commitment to Tarskian theory of truth is not immune to the problem with universal homogeneity of semantics for it simply shifts problem from the level of reference to the level of denotation¹¹. The manner in which Tarski's conception of truth has been utilized might be called into question on

⁶Steven Shapiro, ‘Thinking about mathematics’, New York: Oxford University Press, 2000, p.29

⁷Ibid., p.667

⁸W.D. Hart, ‘Benacerraf's Dilemma’, *Crítica: Revista Hispanoamericana de Filosofía* Vol. 23, No. 68, 1991, pp.87-103

⁹Ibid., p. 90

¹⁰Richard Creath, ‘Benacerraf and Mathematical Truth’, *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition* Vol. 37, No. 4, pp. 335-340

¹¹Creath refers to the Hartry Field's argument that ‘theories of truth involve a reduction of “true” to “denotes”’

yet another ground: while Benacerraf seems to accept that the definition is the only available as a matter of course, after the consideration of the requirements for semantically correct account of mathematical truth, he states: 'I take it that we have only one such account: Tarski's'. On the other hand, Tarski himself defends himself from the question whether his definition does indeed constitute *truth*:

I hope nothing which is said here will be interpreted as a claim that the semantic conception of truth is the "right" or indeed the "only possible" one. I do not have the slightest intention to contribute in any way to those endless, often violent discussions on the subject: "What is the right conception of truth?"

... Disputes of this type are by no means restricted to the notion of truth... and therefore in vain.¹²

It is useful to note that effort of Creath's paper was primarily to win back a solid ground for views that PB collectively labels 'combinatorial', i.e. views which are characterized by the idea of "assigning truth values to arithmetic sentences on the basis of certain (usually proof-theoretic) syntactic facts about them. Often, truth is defined as (formal) derivability from certain axioms".¹³ Thus, Creath alludes to the usage of Tarski's definition of truth to indicate that it might lead to the very same problem author of Mathematical Truth attributes to 'combinatorial' views, namely that the conditions for asserting that the statement is true do not manifest the conditions of the property of correspondence between the language and the fact. Line of reasoning is analogous: just as in formalist approaches conditions of truth reveal mere theoremhood without shedding light on the nature of the connection between theoremhood and truth, presupposing Tarskian semantics does not contribute meaningfully to discussion of mathematical truth as it replaces 'true' with 'denotes' instead.

The epistemological horn of the dilemma also deserves some investigation. Benacerraf's assumes a naturalistic view of knowledge, explicitly stating that he favors causal account: "... For X to know that S is true requires some causal relation to obtain between X and the referents of the names, predicates and quantifiers of S. I believe in addition in a causal theory of reference, thus making the link to my saying knowingly that S doubly causal." It seems that those intuitions might be subsumed under accounts now termed "causal-correlational", but this

¹²Alfred Tarski, 'The Semantic Conception of Truth: and the Foundations of Semantics', *Philosophy and Phenomenological Research*, Vol. 4, No. 3, p. 355

¹³Paul Benacerraf, *op.cit.*, p. 665

point would require a separate exploration. The essential idea seems to be knowledge might be seen as a sort of multi-layered notion, where on the very top there is putative believer with a subjective mental state of believing that S , on the level below we have facts, causally linked to the belief, and at the bottom we find referents of the statement expressing the belief, i.e. predicates, names and quantifiers. On this account it becomes considerably clearer in what way exactly, according to Benacerraf, “standard” view goes awry in accounting for mathematical knowledge. From considerations concerning the semantic constraint, we know that the subject matter of mathematics must be some objects reducible to names, singular terms, predicates and quantifiers in a fashion akin to natural language. Moreover, though PB does not provide any arguments for this step of reasoning and accepts it as a matter of course, it is implied that mathematical objects must be abstract.¹⁴ We also learned from the second requirement that over-all view of mathematics must conform to a general account of knowledge. We additionally established that the account of knowledge that Benacerraf adheres to is a causal one and this, in turn, in conjunction with the supposition that are abstract and therefore causally inert, leads us to the conclusion that on “standard” view mathematical knowledge is unintelligible.

REFERENCES

- [1] Paul Benacerraf, 'Mathematical Truth', *The Journal of Philosophy* LXX 1973, pp.661-679
- [2] Steven Shapiro, 'Thinking about mathematics', New York: Oxford University Press, 2000, p.29-33
- [3] W.D. Hart, 'Benacerraf's Dilemma', *Crítica: Revista Hispanoamericana de Filosofía* Vol. 23 1991, No. 68, pp.87-103
- [4] Richard Creath, 'Benacerraf and Mathematical Truth', *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition* Vol. 37, No. 4, pp. 335-340
- [5] Alfred Tarski, 'The Semantic Conception of Truth: and the Foundations of Semantics', *Philosophy and Phenomenological Research*, Vol. 4, No. 3, pp. 341-376

¹⁴For a review of some arguments see: Hardy (1980) “Benacerraf's Dilemma”