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Process and Action: Relevant Theory and Logics*

Abstract. While *process* and *action* are fundamental notions, in ubiquitous use, they lack satisfactory logical treatment in two critical respects: in analyses of the fundamentals themselves and in logical development. For what treatment they have so far received, under classical systematisation, leaves significant lacunae and induces much paradox. A relevant logical relocation, carried through in detail here, removes such problems, and provides solid ground-work for a satisfactory treatment.

Firstly, as to fundamentals: processes should be explicated, so it is argued, as certain sorts of (time) directed functions (from inputs to outputs); thus they can be represented through certain ordered pairs of relations. Significant logical structures they can enter into are investigated: notably, process lattice and coupled logics, and a generalized category theory (tolerating nonassociativity of composition).

Actions are types of processes, agent-ascribed process. As stock analyses of the differentia, operators and agency, through intentionality, rationality and so on, demonstrably fail, new causal analyses are proposed.

Secondly, as to logical developments: for the most part, the apparently diverse offering of process and action logics to be encountered in the literature are but multiple modal logics: modal logics enriched with further functors of interesting modal sorts. Some, for example, like advertised "process logics" are dynamic logics (themselves basically multiple modal logics) enriched by tense logical functors, themselves modal in character. In a way that is now becoming nonstandardly standard, these modal enterprises can be reworked on relevant logical bases. A main point to such exercises resembles that of other relevant reworkings: namely, the search for correctness, for adequacy to pre-analytic and linguistic data, and therewith removal of paradoxes and anomalies that accumulate under modal analyses.

Logical components from a properly expanded Humean model of action are supplied with relevant logics and semantics, in particular *doing, trying* and *striving, intention* and *motivation*. The difficult question of formalising practical inference is then addressed.

Relevant dynamic logics, paralleling modal developments, are built up piece by piece, relevant theory change is considered within a dynamic framework, and work on relevant temporal and process logics of programming cast, including functors such as *before, during* and *throughout*, is initiated. The present state of logical play is assessed.

I have no wish to espouse, or even give comfort to, a reductionistic philosophy or program. Nonetheless there *are* a few more fundamental notions

*I have been much encouraged in this work by Krister Segerberg, who deserves and has my considerable thanks. It will be evident that his fine investigative work, which I want to see *furthered*, has served as a foil in much of what follows.

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of which almost everything else comparises elaboration or variation. It is an important part of an exact philosopher's enterprise to search out these notions, and to expose their features and roles. Philosophical logic is of course like this, with many everyday argument functors variants upon (though not necessarily reducible to) more basic ones: *but, however, although, though,* upon *and*, to take a simple example. Everyday English exhibits similar variations-upon-basic features everywhere; for instance, most complex terms are compounds of simpler more basic building blocks, often drawn from other languages.

Among the quite basic conceptual notions, in ubiquitous use in both everyday and technical life, are some in the vicinity of *process* and *procedure, doing* and *action*. For long they operated largely uninvestigated logically, and they certainly remain *under*investigated. Only recently have these notions received more exact investigation, and that none too satisfactory. For this investigation, in a classical logical setting, while no doubt enhancing exactness, has unearthed, or produced, many problems in notions that appeared more or less in order as they were. As elsewhere, most of these problems are problems induced by a classical setting (even as modally liberalised); and, as elsewhere, many of them are straightforwardly removed by amending the setting, by relocating to a relevant logic framework.[1] How this happens is shown in part III and IV, where the detailed logical work gets going. Before that the more basic notions are disentangled, and some of their landscape mapped and described, in part I. Then the most basic of them, *process*, is tracked some distance, in part II, and a preliminary theory outlined. These are essential preliminaries to obtaining apposite logics for process and action, as well as for assessing the adequacy of logics reached.

1. Landscape of action and process: significant super- and sub-divisions

Actions are types of processes, agent-ascribed processes. Therein lies a main linkage of action theory with process theory. Therein too lies *one* source of renewed interest in process theory, as an apparent aid to improved, dominant paradigm action theory. *Other* sources to some of which we are bound to revert, are strikingly diverse, but interestingly interconnected; they include the ancient dispute between Becoming and Being, process and substance, Whitehead's theory of process and reality, the theory and metatheory of (biological) evolution, process engineering and control, the theory of computer programs and processing, and dynamic logics, algebras and theories. Each

[1] See Sylvan 91 for information and references.

of these sources, of a specialised or applied kind of process theory, supplies in principle an action theory subdivision.

Processes themselves certainly do not require agents. Actions, however, as distinct from mere happenings and going-ons do require agency, and so agents. At least this is assumed virtually universally in the logical and philosophical literature on action, where furthermore it is normally taken for granted that the agents are one and all (or all bar one) human. We shall certainly abandon the latter chauvinistic assumption, but even the initial agency assumption, which will be retained, can be thrown into doubt. Simply consider the first of many senses paraded in the *Oxford English Dictionary*, which does not (under the *doing* head) require agency: 'I. *Generally* 1. The process or condition of acting or doing (in the widest sense)...'. Examples include scientists entering laboratories or observatories and inquiring of experiments devoid of agents: What is the latest action? (Analogously, the technical term *action* in classical mechanics.) Concise and shorter (too short) English dictionaries regularly equate action just with process; but it is obviously advantageous in more exact philosophy to make the received distinction. Then actions are included among processes — distinguised through agency, or equivalently, agents. How exactly generates some pretty problems.

The main problem with all accounts of agency hitherto is that *too much* is expected of agents: active initiation, production, achievement, success ...purpose, intention ...even rationality. While such features do hold of some agents — purposeful agents, rational agents, and the like — they do not hold generally. Nor should such features be generally required. Not only would it be a departure from what is meant by *action* to exclude familiar examples which do not involve much in the way of these further properties, but as well, to amalgamate action with further features precludes a proper *separate* analysis of the further features, as appropriate to certain types of action (only). Among actions are many casual or even semi-automatic actions, where a perhaps capable agent may not be actively engaged, such as idling, doodling, snoozing, sleepwalking, and so on, and also natural actions, such as laughing, sneezing, and so on. Actions can be roughly delineated through answers to the question *What is so and so* (agent x) *doing*? Answers include natural actions, like coughing, habitual and semi-automatic actions, and even some 'mindless behaviour'. In this respect the present theory reaches deeper than Goldman's detailed analysis of action, which either rules out (p. 18) or substantially neglects (p. 19) such actions, and accordingly does not get down to basics. When basics are reached, features like *intent* can be added (to give Goldman's "basic" act-tokens).

Resolution of a small quasi-technical problem will indicate how undemanding the linkage between agent and doing or process may be. The problem is this: in order to keep in step with the so far developed modal logic of action we need to be able to express answers to the key question *What is x doing?* in the form "x - - -that A", where A is a declarative sentence, and so *that* A is a propositional expression, and the three dashes show relation or linkage. The stock, highly contrived answers to the issues of what this relation (transforming *doing*) amounts to, and how it is to be read, all import much more, in the way of control and intention, than basic actions may exhibit. Consider, for instance, *brings it about, sees to it, makes it happen*, and so forth. These ascriptions are wrong for laughing, snoring, doodling, idling, etc. When x is doodling most often x does not see to it that x is doodling. A less demanding reading that avoids such importations appeals to relations of ascription or creditations; namely: "that A, ascribe the doing to x", or "Credit x that A", later symbolized $\mathsf{D}_x A$.

A similar comparatively undemanding linkage can be seen as relating process and action. Actions are processes whose processing is ascribed to agents, processes *properly* ascribed because say-so works no more reliably here than elsewhere. Moreover, it is ascription or accreditation of the processing of a process that matters. For an agent can start or remotely cause a process without having been said to have done it. The agent may be instrumental in causing it without causing or doing it; but naturally such an agent will not be properly accredited with such a processing. As well, there are further conditions to be met, both on *proper* ascription and on ascription *of* the processing. But what matters for the present is the tandem tracking of process and action: the more restrictive action tracks process through appropriate relation to an agent. Indeed action can perhaps be characterised in that way; process α is an *action* iff, for some agent x, α is properly ascribed to x.

Certainly the similarities between actions and processes are extensive, and worth pursuing. For example, both actions and processes are analytic items; they characteristically break down into components which are assumed to have conveniently tractable features. As actions are supposed to decompose to basic-acts (by Goldman and others), so processes are taken, invariably in engineering, to decompose to subprocesses, which are atomistic operations for which a mechanism can normally be supplied. This is, a process is a composition of a sequence of canonical subprocesses. While such decomposition no doubt works well for significant subclasses of processes, upon which analytic and engineering activity can concentrate, there is no reason to expect that it succeeds in general, that there is for instance an absolute analytic bottom. (Suppose instead that general microprocesses map one-one to an interval of real numbers, or to sets without foundation.)

Much too that has been presented as distinctive of action can be replicated for process. To take an elaborate example which we shall not stop here to investigate, the analysis of the *structure* of action — effected through act-diagrams and level's generation running down, again, to basic sets — made with splendid scholasticism in Goldman, can be reenacted in process terms. (Replace 'act' by 'process' throughout, and allow single agents to be omitted, or supplanted by other synthesizing subjects.) We do not stop to illustrate this structural correspondence because there is much evidence that such analytic atomism is unsatisfactory, except in limited formal settings (settings at which we shall of course arrive, below). The main point lies in the parallels. Many of the more fundamental features celebrated in, or problematic for, *action* are likewise replicated for *process*. In the first place, both are subject to some sort of type-token or generic-individual distinction. The distinction is important in computer-linked logics, where type-processes are represented by (or conflated with) programs. Then each *running of* a given program is a token-process (no conflation on this occasion).

There are also various intermediate rule-governed items in the vicinity of actions and processes, sorts of processes, which share the same sort of duality. These include routines, prescriptions, practices, procedures, which are like programs and may be operated, made-out, or run, but are less mechanistic or non-algorithmic, less systematic in their format, and so on. Some of these have the *big* advantage that although they are typically regulated, although as with routine and procedures there are characteristically operators of some sort, agency is not required. That is, genuine agency, a difficult component for any analysis of action, falls out. So *action* can then be reconstituted, if this isolated agency can be duly captured, as *process* plus *operator* plus *agency*, or — if *procedure* is *process* plus *operator* — as *procedure* plus *agency* (of the operator involved).

A large obstacle in distinguishing action no doubt remains, that of capturing agency, in some sufficient middle-way form (fuller than that of a mere mechanical operator, though not excessively demanding), that alluded to as "agency". Attempts have been made to explain this through — what we have already seen is too much, and what stands even more in need of elucidation — intentionality and, differently (minimal) rationality. According to Cherniak, 'the most basic law of psychology is a rationality constraint on an agent's beliefs, desires, and action: No rationality, no agent' (p. 3)! Differently, von Wright answers his question 'What is action ?... Action is normally behaviour understood, "seen", or described under the aspect of intentionality, i.e. as meaning something or as goal-directed' (83 p. 42).

According to Segerberg, what makes the difference as regards action is *intention* (or *goal*, but elsewhere he is prepared to substitute *agency* or *ability*, and in some places he appears to backtrack on this differentiation theme). According to Pörn (p. ix, as if responding to Goldman who, stuck with a wants-beliefs model, was worried about agents' doing something *else* they do not want to achieve what they want), '...it surely is the case that [even] if an agent acts in order to do something else, he does what he does intentionally'. But that wrong direction is about as far as Pörn is prepared to go towards conditions on agency; he ends his extraordinarily dense tract by deliberately avoiding such critical issues concerning agency as what counts as an agent (p. 123)! More common than abstention is excess.

Though both intentionality and qualified rationality show promise, as general features of *certain* agents (intentional and rational ones), as features that *such* agents should or must sometimes (perhaps even normally) exhibit, neither operates as a feature every example of action has. In showing that neither succeeds, much of previous action theory is subverted. The effect of subversion is not however dramatic. A main reason for this is that required features of action such as intentionality were rarely well specified; they functioned as logical black boxes, and theories that required intentionality could equally well be geared to χ-ality with but few constraints on χ. In fact intentionality itself is rather like that. Look at *intention*, which is sometimes explained as very complex, 'broadly involving mental application or effort', *mental action* to expose its deep circularity. To make matters worse still, intentionality has often been introduced in order to explain what is explaining it, *mental* features. Such a problem seriously afflicts (behaviouristic) action theory, which has been promoted as a way of accounting for mental features of actors.

Consider, to get to gritty details, the four-fold classification generated by the contrasting pairs: intentional-unintentional and rational-irrational. There may be more classes than these four, i.e. rational-intentional, rational-unintentional, irrational-intentional, irrational-unintentional. Perhaps allowance should also be made for nonintentional actions which are neither intentional nor unintentional (cf. Segerberg 84, p. 75). Further classes would only improve our case against normal requirements for action. Now examples appear to show that all four classes distinguished are exemplified. The intentional and rational class of actions is commonly, though perhaps erroneously, taken as providing *normal* actions. Among intentional but irrational actions are many cases of neurotic and psychotic actions. More problematic are unintentional actions. While rational (economic) action can evidently have unintended consequences, it is less clear that it can be unintended.

But let us work up from cases where agents, though proceeding carefully enough, produce unintended damage. For instance, an agent causes a vase or bottle in a market to fall and break, perhaps by brushing it or walking by and thereby vibrating a pile it is in, or the agent causes some people to be put out of work, or .... Then we should normally have no hesitation in claiming that the agent broke the vase, *did* that, though unintentionally. Similarly, that agent, through his rational accounting practice or whatever, did that, put those people out of work, though unintentionally. (There is a logical principle operating here, of the approximate form: x does A, A (directly) causally implies B/x does B.) For irrational unintentional actions, replace our rationally-proceding agents by irrational actors. A psychotic agent, while locked into an irrational behaviour sequence, does something unintentionally; for example, while emerging as Napoleon, brushes a vase and thereby causes it to break. No doubt such examples are not presently commonplace, no doubt they are irksome for theories that aim to couple responsibility or the like to action rather than to certain subclasses of action. Nonetheless they may occur and (prima facie) put paid to influential accounts of what constitutes agency.

An agent is not always an agent. Agent types are sometimes caught up in happenings where they do not exercise agency. For example, if an agent is used as a missile or as a battering ram by other (despicable) agents, then the sometime-agent exerts no relevant agency. Similarly when the sometime-agent is unconscious. Borderline cases here include such procedures as sleepwalking. Thus, as a minimum, an active agent presumably must be not merely alive, conscious and to some extent *aware* of what is going on, but *exercising* some *causal* role in what is happening. As exercising a causal role appears to imply having a certain awareness, which in turn implies the other necessary conditions mentioned, causal efficacy looks like the appropriate place to look. Conveniently it also coincides with noncircular dictionary accounts of what is an agent (circular ones give 'one who acts' or the like) in terms of 'exerting power' or being 'the material cause'. As well it delivers us back to more or less where we started, in terms of active agents initiating or originating processes with which they are credited, and aligns us with what visionary logicians or poets have proclaimed, that an agent *makes* things happen, causes processes, instead of passively suffering external causes, the slings and arrows of outrageous fortune. It is plainly not enough that an item x is an instrument in the causing of A, that x starts a causal chains proceeding to A, or even that x causes A, because nonagents can produce some or all of these A-outcomes. One way of ensuring that x is not merely a passive part of a causal chain, or an instrument or cog, but exerts relevant power, is to require further that x is not caused to cause A.

An initial stab then at defining agency begins as follows: x is an *agent* as regards A iff, for some B directly pertinent to A, x causes B but x is not caused to cause B. Among the evident difficulties with such a definition, apart from the matter of explaining pertinence, are those generated by determinism, under stronger versions of which no subject is not caused to cause what it causes. Where such determinism stands up (under relevant logical theory it does not, so FD argues), an alternative way will have to be found to say what is intended. Similar pseudo-difficulties flow in from causal theories of action, such as that floated by Davidson (defended in Bishop), which suppose that action is always a causal process, agents being caused by their mental conditions, and so, these conditions being caused to cause what they cause (but such theories depend both on determinism and on semantical skulduggery regarding causation: see FD). Less evidently, the putative definition is vulnerable to counter-examples. A (weak) example is that of a robot programmed to cause a sequence of happenings but with a randomiser built into the program, a robot reflecting a primitive idea of free-will. But then, as well as sometimes not causing what it is presented as an agent regarding, it fails the anti-determinism requirement. Once *agency-regarding* is defined, other neighbouring notions are readily defined. Thus x is a *sometime-agent* iff, for some A, x is an agent as regards A. *Agency* itself is a characterising attribute of sometime-agents, defined by abstraction. And so on.

Although but a *modest* account of agency has been sketched, the more that is often urged, further essence to fatten up the account, is not essential, and is too often both excessive and restrictive. Such a point applies, in particular, against the Hume-attributed wants-beliefs apparatus that is frequently introduced as a supposed prelude to causation.[2] Even if it is hard to point to actors devoid of wants, outside the (controversial) confines of Eastern religions, it is not difficult to conceive of subjects of this sort (the lower appetitive part of the Greek soul has atrophied, or been surgically removed). Motivation does not require wants. Nor does decision, a modelling

[2]Although it is regularly referred to as "wants-beliefs model", more strictly it is wants-belief-causation model (see Goldman e.g. p. 223). And as often presented it requires still more logical apparatus: see part III.

Considerable liberty has taken in ascribing this apparatus to Hume (philosophical superstars need lots of credits). Hume proposed no model of action, but at most a theme concerning *motivation*, which is itself different from what it is often construed as asserting. What Hume claimed is that reason alone (not belief) is never a sufficient motive for (chosen) action, but must be coupled with passion. (He did not contend that we are motivated entirely by desire.) The theme is perhaps more plausible as regards motivation, and also explanation, than it is as stretched and warped to a theory of action.

for which can function with values and without wants, and with probabilities and without beliefs. A philosopher's god or a sage can be an agent without being a Humean agent (or a human agent, Humeans suspect).

Therein lies one significant reason why the enterprise of explaining all action through wants-beliefs apparatus (or less), of forcing the whole of the behavioural or social sciences into this narrow mold, fails. Certainly such a grand reduction enterprise continues to flourish, particularly in Scandinavia. A sustained attempt to push (a core individualistic part of) such a reduction through may be found in Goldman, who endeavours to display all practical inference, including decision and deliberation, in wants-beliefs terms and to show that one and the same model is adequate for explanation in the behavioural sciences.[3] Certainly, too, linguistic forcing goes on in this enterprise. For one thing, *want* is generously redefined to make the procedure seem to run. For another, reduction takes (like soft determinism) a compatibility form; other inference and explanation schemes are compatible with a want-belief surrogate. But, short of infiltration of wants and beliefs that aloof unselfish agents may not have, this looks implausible. Contrary to the prominent run of reductionistic theories, there is no solid reason to suppose that such a *single* narrow account of action (including decision, deliberation, inference) can succeed everywhere. Pluralism is, here elsewhere, a much more likely scenario.

Approaching action from the more general setting of process is particularly illuminating in this respect. For processes which do not require agents at all are evidently not amenable to wants-beliefs reductions or variations thereupon. That includes such important process as abstract inferences, and so mathematical operations, which are independent of agents (and operators). Nor does a wants-beliefs apparatus automatically enter into what distinguishes actions among processes, proper accreditation of agents. The substantial deviations of some agents from wants-beliefs modes of conduct means that even when agents do enter, reduction is not to be expected (though *simulation* with "wants and beliefs" cut loose from their moorings is not excluded). Philosophers' agents are but a subclass of agents (desire- and anxiety-ridden agents, orientals might well say), and Humean actions are, like human actions, a quite proper subclass of actions.

A Humean wants-beliefs reduction is not the only way a reduction may proceed. With the veneration of all things computational and mechanical, including therefore computing machines, there is some metaphysical enthusiasm abroad for what amounts to a very different double reduction: of

[3] See e.g. p. 223 and esp Ch. 5. The route the grander project, encompassing social science, presently takes can be glimpsed in Pörn, Chs. 4 and 5.

everything to processes (in line with the process tradition in metaphysics from Heracleitus through Whitehead) and of processes to programs, computer programs. (Some computer afficionados are reluctant to go quite so far, preferring to retain some hardware to run the programs, extensionalised Nature?) While the whole enterprise is fraught, like all crazy reduction programs, with difficulties, it does have several entertaining features. One starts from the analyticity that programs entail programmers. Programs for natural processes, up to and including of this universe, require then the introduction of super-programmers, such as God or Nature (now highly intensionalised), as no lesser "beings" can accomplish the task. But under the full reduction, God itself is a further category of programs, which requires a further programmer, which ... (who programs the last programmer at any stage? Is the "last" programmer that impossibility, the complete self-programming programmer? Else, how does anything at all get programmed, under this apparent vicious regress? There are only extremely awkward alternatives.)

2. Towards a general theory of processes and procedures

So far we have been primarily engaged in distinguishing actions among processes. We turn now to the grander enterprise of trying to characterize and to capture logically processes themselves. This is no small and easy assignment.

Processes are ubiquitous items, not merely in daily practice and experience, but in theory as well. Many of the main topics upon which logic and epistemology concentrate concern processes: for example, argument, inference, perception, cognition, reflection, choice and thought. Outside philosophy, processes are at least as prominent, in practice as in prefigured theory, especially in computing science and engineering, but also in biological and social sciences. Information processing is a major part of computing, main units that accomplish it being processes. New disciplines such as process engineering, and their more innovative branches such as bioprocess engineering, are largely taken up with various types of processes, and with the institution, control and regulation of them, systems design for them, and so on (thus, e.g. upstream and downstream processes, process automation, measurement and control of processes, mass and energy conversion processes in bioreactors, etc.). But though the theoretical application of the notion of *process* is ubiquitous, there is so far no viable general theory of processes.

It is unlikely that engineers and technicians in the special sciences will produce such a theory. Theories of the most general sorts — such as those of objects and properties, processes and sets, are commonly left to philosophers, exact philosophers for more exact formulations — and are bound to remain of concern to them, even if parts of them are expropriated by other overlapping disciplines such as mathematics, as has happened with set theory. When well chosen, these theories have many applications, both technical and philosophical. Process theory too can afford such a synthesis, with wide and deep applications.

Certainly there have been previous attempts at furnishing a general theory, the most conspicuous being Whitehead's effort, culminating in his *Process and Reality*. But Whitehead's actual treatment of processes is disappointing. It is very brief, a small part of a large book, and it lacks generality. Whitehead proceeds almost immediately into two rather special types of processes, transition and concrescence. These types, though certainly of importance, are far from exhausting the range of processes of interest. The processes Whitehead considers are, moreover, always *time-ordered* processes. There is no doubt a basis for such a temporal restriction on the everyday notion of process, which is normally so restricted; but for a general theory, and to accommodate all processes, it pay to relax this restriction, and to generalise on orderings admitted. In fact this indicates the strategy to be pursued; namely, to introduce, what is technically more amenable, a generalised notion of process, and then to cut down to the normal notion.

Moreover, once appropriately generalised, there is nothing especially difficult about the ubiquitous notion of *process*: a generalised process amounts simply to a certain sort of (time) *directed function*. Processes stand to relations, to many-valued functions, then, as vectors stand to scalars; they superimpose a direction, an ordering relation. But though the basic idea is simple enough, the notion, like that of set and category for example, is fundamental.

For processes are what they are and not something else (for instance, what they may be *represented* as, an ordered pair of relations). That Butlerian theme does not of course imply that processes cannot be partially characterised, for instance roughly, but informatively, as items that happen, or differently and more informatively, axiomatically. Processes, like events and (mathematical) categories, are a kind of item (and so appropriately studied in item-theory, i.e. generalised object-theory). They are items that typically *go on, happen*, over a period of time (cf. von Wright). So immediately there are two coupled components: a relation or function (from domain to range, or input to output) and a vector or interval (of time). Accordingly, items which simply relate, as connecting inputs with outputs, are not processes per se; for instance, to take linguistic examples, connectives, functors and so on, are not processes. A time-like element, indeed a *froming*, is essential.

But it is, on its own, not enough. One event's happening after another may not constitute a process. Thus too processes are not reducible to relations of events. Though processes admit of analysis into ingredients, they do not admit of, or need, reduction or elimination.

Processes are certainly not substances or persistent things. Accordingly there is no ontology of processes to try to remove. Existence is not a category that properly applies to processes, in the way it does to things. Processes go on; they do not significantly exist, or not. In a two-valued (or four-valued) setting, we may say that processes do not exist. But processes do not exist in the sense that they are the wrong sorts of items to which to ascribe existence. Nor do processes, as a specific type of item, facilitate adequate reverse reductions (as proposed by Whitehead, and as inferrable from category theory). Reverse reductions, of (certain) objects, to processes or categories, are artifices. Nevertheless, it may be technically interesting that they can be achieved, in certain limited contexts.

Factuality is likewise not a category that properly applies to processes. Rainfall, or the falling of rain, is not significantly a fact. Surprisingly, then, von Wright has tried to explain, appealing to just such processes, how processes, events and states-of-affairs are three types of facts (63 pp. 25–27). But there is here real conceptual confusion, amounting to category mistakes (in von Wright and repeated in Segerberg 90), in the contention that all those items are types of *fact*. A process is not *any* sort of fact; its *taking place* may be. For it is no doubt a fact that some processes, or types of processes, occur; it is a fact that particular processes take place. To keep up with our guides, yet skirt this conceptual bog, we enlist the term *factive*. The sorts of items whose *happenings* are sometimes facts are accounted factives.

Among *factives* — doings, happenings and occurrings — are processes, events and state-of-affairs. These items are main ingredients in a general theory of action, as well as in much other theory, such as probability theory and statistics. A nonreductive object language rich enough for such a general theory will include terms for each of these sorts of factives. Of course reductive theories of action — a main present style — try to reduce some of these sorts of factives to others, or even to eliminate them all.

While there are good grounds for resisting those practices, largely pursued on bad ontological grounds, there are sound reasons for grouping factives together. Not only are they closely interconnected, processes for instance going through states; also they have similar structural features. As generalised, factives share a basic algebraic structure. However this structure is not, as is usually imagined, a Boolean algebra, or an extension thereof, but rather a De Morgan lattice. One reason for this springs from the paradoxical features of Boolean algebras of factives. Contrary to Boolean pronouncements, $\rho \cup N\rho$ and $\sigma \cup N\sigma$ are not the same generalised processes, equivalent to a unique "universal process" 1. A basic algebra of generalised processes is given by De Morgan lattices; analogously too for other types of factives. Such a lattice **DM** in a class of processes $\mathcal{M}$ (closed under the operations involved) is a structure $\langle \mathcal{M}, \cap, \cup, N, \leq \rangle$ where $\cap$ and $\cup$ are join and meet operations on $\mathcal{M}$, N is De Morgan negation operation on $\mathcal{M}$, and $\leq$ is a partial order on $\mathcal{M}$, all subject to De Morgan lattice postulates (displayed e.g. in RLR p. 183). Consider N, the operator which separates these structures from straightforward lattices. Where ρ is a process in $\mathcal{M}$, so is $N\rho$, such that $NN\rho = \rho$ and when $\rho < \sigma$ then $N\sigma < N\rho$ for ρ and σ in $\mathcal{M}$. Observe that unless processes are generalized, formalism (or closure) rules like that for N (and analogously $\cap$ and $\cup$) are in serious doubt, since otherwise $N\rho$, the negative of ρ, may not always count as an ordinary process.

These De Morgan lattice structures yield De Morgan lattice logics of processes. A first unfamiliar form DML for such a logic is a derivational one, formulated using the meta theoretic auxiliary $\vdash$. The statemental form $\mathsf{A} \vdash \mathsf{B}$ informs us that formal object A, a compound process in $\mathcal{M}$, yields formal object B, also such a process. The postulates of process logic DML is as follows, with Γ and Δ finite (nonnull) sets of formal objects:

Axioms:	$\mathsf{A} \vdash \mathsf{B}$		
	$\mathsf{A} \cap \mathsf{B} \vdash \mathsf{A}$	$\mathsf{A} \cap \mathsf{B} \vdash \mathsf{B}$	
	$\mathsf{A} \vdash \mathsf{A} \cup \mathsf{B}$	$\mathsf{A} \vdash \mathsf{A} \cup \mathsf{B}$	
	$\mathsf{A} \vdash NN\mathsf{A}$	$NN\mathsf{A} \vdash \mathsf{A}$	
Rules:	$\Delta \vdash \mathsf{B}$	$\mathsf{B}, \Gamma \vdash \mathsf{C}$	/ $\Delta, \Gamma \vdash \mathsf{C}$
	$\Delta \vdash \mathsf{B}$	$\Delta \subseteq \Gamma$	/ $\Gamma \vdash \mathsf{B}$
	$\Gamma \vdash \mathsf{B}$	$\Gamma \vdash \mathsf{C}$	/ $\Gamma \vdash \mathsf{B} \cap \mathsf{C}$
	$\Gamma, \mathsf{A} \vdash \mathsf{C}$	$\Gamma, \mathsf{B} \vdash \mathsf{C}$	/ $\Gamma, A \cup B \vdash \mathsf{C}$
	$\mathsf{A} \vdash \mathsf{B}$ /	$N\mathsf{B} \vdash N\mathsf{A}$.	

The logic can be represented in more amenable equational form, with $\mathsf{A} = \mathsf{B}$ read, A is the same object as B. $\mathsf{A} = \mathsf{B}$ is defined as $\mathsf{A} \vdash \mathsf{B}$ and $\mathsf{B} \vdash \mathsf{A}$, that is through two way deviation. Conversely $\mathsf{A} \vdash \mathsf{B}$ amounts to $\mathsf{A} \cap \mathsf{B} = \mathsf{A}$. Such a purely equational formulation may be developed entirely similarly to that for Boolean algebra equational logic (as carried out in RLR pp. 118-120). A more familiar rewriting of derivational logic DML is propositional (the form developed in RLR pp. 104–11), the logic being propositionalised by dummying in a *happens* functor ***H*** or equivalent. The logic so construed coincides in turn with the relevant logic of first degree entailment (in connectives $\{\&, \vee, \sim, \rightarrow\}$ with no nesting of $\rightarrow$; see RLR pp. 171–2 for details of the logic and interconnecting theorems). A considerable theory can now be developed upon process structures by direct analogy with developments upon event structures in theories of probability and statistics. For example, a beginning is made by introducing probability measure functions on De Morgan process lattices. Thereby opened is a direct and improved route into much of what is said in statistical theory of various types of processes (Markov, point, etc.), there typically reduced to sequences of events.

As the basic algebras and logics introduced appear isomorphic for different sorts of factives, they afford but little help in distinguishing the items. What does? The different sorts of factives are distinguished, in the first place, through the classificatory predicates that they significantly admit. Among such predicates are these: $\boldsymbol{G}$ for 'goes on', 'obtains', $\boldsymbol{H}$ for 'happens', 'takes place', as well as such descriptive predicates as 'is an event', 'is a process' and 'is a state-of-action', Within the broad confines of first degree entailment, $\boldsymbol{H}$ enjoys homomorphic features: for $\boldsymbol{H}(\rho \cap \sigma)$ iff $\boldsymbol{H}\rho$ & $\boldsymbol{H}\sigma$; $\boldsymbol{H}(\rho \cup \sigma)$ iff $\boldsymbol{H}\rho \vee \boldsymbol{H}\sigma$; $\boldsymbol{H}(N\rho)$ iff $\sim \boldsymbol{H}\rho$; and $\rho \leq \sigma$ iff $\boldsymbol{H}\rho \rightarrow \boldsymbol{H}\sigma$, for ρ, σ in $\mathcal{M}$. It is these features that are used in converting formal object to propositional logics. Similar properties are normally expected for $\boldsymbol{G}$. No doubt some of these predicates can be defined, or approximated, in terms of others. For there are intimate interconnections between these factives. For example, the beginnings and ends of processes (where there are beginnings or ends) are events, as are intermediate "points". So, in a weak sense, a process comprises a sequence of events (as differently of states). But an eliminative account cannot be reached in that way, because a process is a holistic item, not a mere string; a sequence-of-events account would leave out the crucial relational glue that holds natural processes together, and may indeed fail to distinguish different processes issuing in the same strings of events, and vice versa.

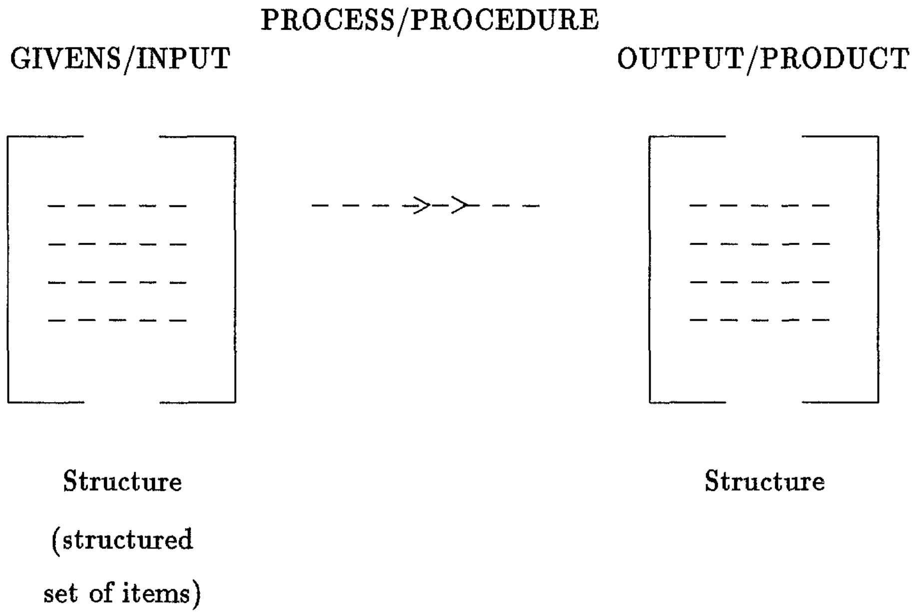
Events themselves are not normally regarded as processes, as they do not go on suitably, but happen and are finished. But no doubt under a different stretching of the term *process*, events may be encompassed, as point or short duration processes. It is the reverse reductive procedure that is, by contrast, particularly problematic. For more than nonnull sequences of events, *paths* as they are sometimes called, are required to begin to represent ordinary processes. Again there are their interrelations to adjoin. There are identity criteria to sort out and get right. For example, one and the same whole process may have significantly different path analyses depending on the breakdown made. Path analysis, if taken as offering more than a partial representation (unfortunately what happens in so-called process logics[4]), is liable to go astray, representing what is the one process as several different ones, a matter that is made worse when competing atomic components are presumed, for instance, states or worlds instead of events. The problems with trying to represent long or continuing processes in terms of sequences of their constituent atomic factives such as mini-processes, or surrogates for these such as events or static states, resembles analogous problems in trying to treat persistent or static items as strings of their individual time-slices (for details and proposed resolution see JB pp. 393–4). Resolution can proceed in each case, whether confronted with change over time or on-going processes, by dumping atomistic reduction and resorting to a certain holism. Under this processes are treated for what they are, more holistic items expanding (often sausage like) over time, not always atomistically (like beads of events on a string). Fortunately there are now more promising alternative approaches, elaborating on available theory. One is by way of a theory of procedures, developing out of computer procedures. Another, still more attractive, looks to category theory; for this theory, rather exceptionally within available mathematics, can be interpreted as offering a holistic treatment of certain items.

Techno-logical fundamentals for procedures and processes

A ready way to approach processes is through procedures, upon which decent technical progress has already been made (e.g. Rennie and others). Stock examples of procedures are applying algorithms or following rules, for instance the computing of a function according to an algorithm, as represented in turn through a program of rules or commands for the computation. Generalising involves removing rule-governing presumptions and relaxing effectiveness, exactness and other requirements. Given general procedures, processes can be reached by abandoning further restrictions, in particular such requirements as that there is some agent, facilitator or controlling operator. An initial characterisation of what is supposed to count as a logical *procedure* can be obtained from wider definitions of 'deduction' or 'derivation' commonly encountered: *to proceed logically* (for the operator concerned) is 'to derive or draw a result from something already given'. A *procedure*, more generally, is a *froming*, of some operator (or agent) controlled type, from one structure to another. Usually of course this overall *froming* will be broken down into a sequence of steps, each applying some definite rule. But more generally a process will transcend such analytic limitations, along

[4] A nice presentation of the basics of this theory is given in Segerberg 90 pp. 11–13 and 84 pp. 79–81. "Process logics" themselves reappear in relevant garb in part IV below.

with shedding operator control; if you like, a general procedure is a proce/ss with operator control (-dure). An initial systems diagram accordingly takes the following (closed) form:



Or, in condensed notation, which at once represents the systems diagram in much more familiar shape: $[A_1, \ldots A_n \ldots] \rightarrowtail [B_1, \ldots B_m \ldots]$, or $\Gamma \rightarrowtail \Delta$, where Γ and Δ are structured sets of items, and $A_1, \ldots, A_n, \ldots, B_1, \ldots, B_m$, $\ldots$ are such items, the square brackets representing the structure. A non-trivial extension of importance allows Γ and Δ to be null. In this way we can allow for non-terminating processes, these have no start state or no stop state.

Put differently again, a process is a multiple-value map on structures, a structure projection. Like all such maps it is a relation of a certain sort, an active, a directed relation which, as with any process, suggests movement or change (and so a time-factor). But though some of these further features of processes could be represented, through tensing especially, it is advantageous to abstract beyond them.

A process can be seen then as comprising two parts, just as suggested by its etymology *pro/cess*, forward-gone or -going; it consists in *a directed relation. It thus comprises both a relation or function, a going or fro(m)ing* *of some sort, and an order or direction, forward.* It differs from a mere (static) relation much as a vector differs from a mere interval, in involving a direction. It is a relation *with* an order, in most everyday examples, a time order. It is something that can be approximated by a *many-valued function* insofar as this is seen, by contrast with a relation, as having an ordering written into it.

The trouble with many-valued functions on their own, and even more with relations (especially as assigned contemporary extensional representation), is that they tend to shed crucial dynamical features and become static objects, set-theoretic scalars. But what are sought in process theory are dynamic relations — relations with a flow, temporally-directed relations in the usual special case, directed relations in the general case — not action-frozen relations. Much as scalars are vectorised by adding a direction, to obtain directed intervals, so relations are processized by adding a direction.[5] Processes, as directed relations (or better, directed many-valued functions), may accordingly be expressed vector style, $\rightarrow_f$ where f is a function. In standard extensional representation, which will be largely avoided, a process is then portrayed as an ordered pair $\langle f, \rightarrow \rangle$. To emphasize the direction, the arrowing, an inversion of the vector notation will normally be preferred; so $\rightarrow_f$ will be reexpressed upside down $\rightarrow^f$. The notation itself thereupon gives a part of the main technical game away to those familiar with (mathematical) category theory. The category connection will explain some features which may have been puzzling, such as the apparent dispensibility of the structures to and from which processes proceed.

To approximate categories we shall introduce "identity processes" where nothing happens. For all our eagerness to adhere as closely as feasible to ordinary scientific usage of the term 'process', we stretch the term to include certain degenerate "processes", such as static cases where nothing goes on (except the background hum of this universe). Normal processes will then be non-degenerate process. Among degenerate processes, which would not normally be accounted processes, are all those myriad static examples of things remaining as they are, such as von Wright's typewriter standing on his desk over some interval, as well as identity programs perhaps differently routed where the output coincides with the input. To reiterate, then, the

[5]Some of the points were suggested by a reading of Bohm's introduction (cf. p.9). In proportionality notation:

$$\frac{\text{Process}}{\text{Relation}} = \frac{\text{Vector}}{\text{Scalar}}.$$

The analogy has its limits, because a relation has a direction of its own, that of the ordered pair at extensional bottom. But the direction of a process may vary independently of this, consider, e.g., that most immediate of cases, a relation evolving through time.

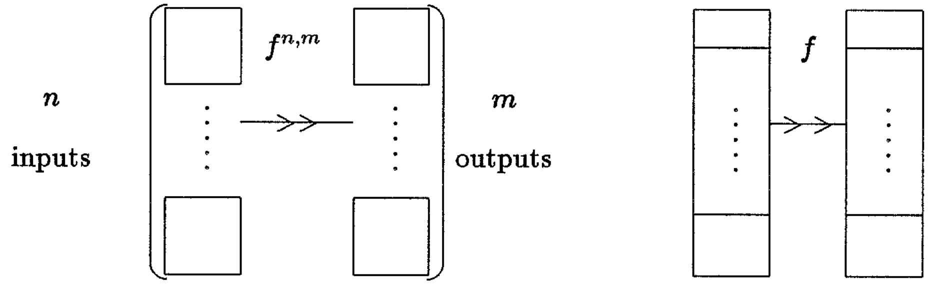
main strategy is really this: to *expand* the notion of process — to super-processes — to include what would not normally count as processes, and then to effect a *cut-down* — within directed functions — to reach processes proper. For the present something like the *happens* predicate $\boldsymbol{H}$ and the *goes on* predicate $\boldsymbol{G}$, taken in combination, can effect the cut-down. For what have to be peeled off are degenerate processes (which do not happen), events (which do not go on), and generalised processes where the orderings are not an appropriate time orderings (and so satisfy neither predicate).

Though such technical strategems are resorted to, no reductions are attempted. Many however are the proposals or attempts to dispose of either processes or of objects, or to reduce one to the other. These attempts are by no means confined to philosophical regions. The recent history of mathematics has seen a swing from set theory, which carried initially a reduction program of all mathematics to its objects, sets, towards category theory, which in purer form is linked with a reduction theme for all mathematics, including set theory, to functions.

An all too typical approach to processes and their theory is to fix upon restricted type of process with desirable features (e.g. it is mathematically amenable in terms of recently elaborated theory), to develop a non-process representation of such processes, and to declare or presuppose that processes in general are thereby captured. The original procedure of Whitehead, while it certainly did not make the common mistake of presuming that a set- or relational-theoretical representation of processes is adequate, fell into the error of restricting processes to a couple of sorts, without doing anything (what would presuppose a more general theory) to show that these sorts are suitably canonical and thereby suffice for the recovery of the full theory. But many successors of Whitehead do go the whole erroneous hog, presuming also a satisfactory set-theoretical representation. An example is the definition of processes as labelled partially ordered sets of a certain sort, a definition generalising upon set-theoretic representations of *derivation trees* of context free grammars (see Maggiolo-Schettini and Winkowski pp. 255–7, p. 245). While such derivation trees are no doubt (canonically) linked to significant sorts of processes, many processes are not of this sort, for instance laying an egg, losing leaves, lifting a stone. Nor are most squeezed easily or at all into this narrow (linguistic) form. The proposal is distinctly worse than that favoured representation of propositions as sets (of sentences, or of complete descriptions). Processes are regarded as no more than their descriptions, which are then accommodated through set-theoretic linguistic theory. Neither element should carry conviction. Even simple processes like routines may defy easy or very satisfactory description. Humpty-dumptyism, redefining already established notions to suit some ulterior local purpose, is no doubt extremely fashionable in symbolic enterprises. But it has had extravagantly extensive field days; it is time a halt was called. Processes — which are not sets of any sort — make a fine place to stop.

Though in older usage *process* and *procedure* were often simply equated (see e.g. OED), there is point in distinguishing them, in a way suggested by, but no doubt refining, contemporary usage. In these terms, already adumbrated, *a procedure involves*, at least implicitly (as supplied by the context), *an operator*, agent or controller. The controller of a procedure need not be an agent in any narrow sense (e.g. as understood in action theory), but can be a computer, a robotic arrangement, or simply a programmed machine. Natural processes, however, such as earthquakes or volcanic eruptions or coastline erosion or subsidence, are not procedures (except on certain cosmologies, where sometimes they are actions, e.g. of angry gods or spirits). Nor are spontaneous processes, nor random processes, procedures. Processes thus form a more comprehensive class than procedures. Every procedure involves a process, but procedures reflect some sort of operator, agency or control (whether that of a computer or an insect) which processes need not. The explicit symbolisation for a procedure adds to that of a process, $\rightarrow^{f}$, appropriate notation for an agent a, and so takes some such form as $\rightarrow^{fa}$, $\rightarrow^{f}[a]$, etc.

Naturally, procedures and processes — that is - - - -*ing from* one structure to another — come in many different forms and sorts, depending on how the process verb blank '- - -' is filled out, i.e. on the map(ping) involved. These can be distinguished where necessary, for example symbolically in the fashion of chemical reactions and category theory by superscripting the arrow with predicate letter, which relevant features of the relation involved (e.g. that the main catalyst was heat, that the function was a certain isomorphism). Such expressions as $\rightarrowtail^{f}$ with the (two-place predicate) symbol f intended to represent a relation, such as that between the input and output of an economic process or a chemical reaction, raise questions as to requisite generality of the representation, in particular whether higher place predicates linking multiple inputs and multiple outputs should not be introduced. But so long as inputs and outputs can be any structures, higher place connections add nothing that cannot be accommodated; for instance, a multiple input is simply a single more complex input. And generally can be represented



Another major figure in the development of process theory, Curry, makes the requisite point for logical sequents, a type of procedure, and also as regards the special case of *effective* processes (see p. 38). A sequence of inputs simply amounts to a sequential input.

Certain assumptions are usually made about the processes and procedures to be investigated, in particular that they are not merely one-off, but like experimental procedures, in a fashion repeatable (in replicable conditions). The idea that processes are unique and unrepeatable has been a serious impediment to the formulation of a process *theory*, as to the development of satisfactory account of *reasoning*.[6] As with logical theory generally, such obstacles are surmounted by operating with types, rather than tokens which are unique and unrepeatable. These features of tokens obviously do not extend to types.

For much theory it is important to restrict generality of type, to drop down to more specific sorts of processes. Certainly further conditions must be imposed in order that the procedures fall into such prized classes as *logical* (whether correct or incorrect), *rational, reasoning, causal*, etc. Moving a barrow of bricks from a loading point a construction site is a repeatable procedure, which does not fall into the intended classes, though it may meet other standard conditions imposed on procedures, for instance conforming to or following a rule, a stock procedure much bruited about in philosophy.

The sort of intellectual procedures envisaged in the theory may involve an unenergetic physical component, as in drawing a marble from an urn, or shifting a piece on a gaming board. They are typically *armchair* procedures which are *rule following* or governed.[7] And they include the following sorts

[6]For an striking example of this blockage in effective action, see Angell on reasoning.

[7]Many subclassifications of logical and philosophical importance suggest themselves

of processes: inference, derivation, detachment, selection, sampling, decision, rearranging (a configuration). The idea is of course that the theory of logical procedures should include everything that (reasonably) gets accounted logic in a generous sense, including, in particular, such subjects as decision theory, statistical inference, and parts of computing theory and sampling theory, as well as analogical inference, lateral thinking, abduction and so forth. To be sure, the main focus of the logical theory would no doubt be upon certain *reputable reasoning* procedures such as x-ductions of various approved types (with x for *de, in, ab*, etc.) A main intended application of the theory will thus shift away from the more conventional temporal processes and humdrum effective processes, to intellectual processes, reasoning processes especially.

Before turning away from the projected more general theory to specific types of processes or to logical developments, it is worth showing that there is already a general mathematical theory, of suitable holistic sort, which will admit of extensive elaboration. A generalisation of a category theory, "semi-category" theory, allows *one* elegant representation of certain structures of processes.

Process theory as generalising mathematical category theory

A distinction straightaway emerges between *one-step* procedures, of which weeding a garden and (usually) making decisions are examples, and *many-step* or iterable procedures, of which game playing (up to an end point) and logical argument are cases. For iteration, with processes $\Gamma \to \Delta$ and $\Theta \to \Sigma$ in that order, a minimal condition is that Δ and Θ overlap in sort.

It is assumed that the iteration of procedures is a procedure, i.e. that procedures, and more generally processes, are closed under composition. In particular, then, given procedures $\Gamma \rightarrowtail^{f} \Delta$ and $\Delta \rightarrowtail^{g} \Theta$, there is a composite procedure $\Gamma \rightarrowtail^{f \cdot g} \Theta$, with $f \cdot g$ called the (*simple*) *composition* of f with g. (More complex composition would combine f with g from processes $\Gamma \rightarrowtail^{f} \Delta$ and $\Theta \rightarrowtail^{g} \Sigma$ where $\Theta \subseteq \Delta$ for an appropriate notion of structure inclusion.)

Among initial procedures, identity procedures, $\Delta \rightarrowtail^{i} \Delta$ with identity map i_Δ, can always be included. These identity maps will naturally have the correct properties under composition; for f mapping to Δ and g from Δ,

at this stage. For instance, among rule governed procedures, there are those where the rules are effective (and so the procedures are effectively determined) and those where they are not. Marking out effective procedures is an important matter to which we shall elsewhere return — with a view, in particular, to assessing the prospects for a more definitive argument for the Church-Turing thesis. Less intellectual nonlogical procedures include, for instance, perception of various sorts.

Id. $f.i_\Delta = f; \quad i_\Delta.g = g.$[8]

While identity "processes" are evidently degenerate processes where nothing happens (unless for instance composed of a change and its cancellation), they are theoretically advantageous and presumably can be adjoined conservatively.

As may be evident, process theory so elaborated, is fast encroaching upon mathematical category theory, *kategory* theory as it will be pedantically called here.[9] It does not however reduce to kategory theory, but generalises upon it, essentially in two ways:

Firstly, it is always assumed, at least in exposition and interpretation of kategory theory[10], that the arrows of kategory theory represent functions, whereas the corresponding processes of *processory* or *faktory theory*, as we shall call it, are *many-valued* maps, not always (single-valued) functions. Consider the process of baking in manufacturing a cake; sometimes the output is a success, sometimes, with the same inputs, a flop. Or, for those allergic to multiple causation, consider casting a die or tossing a coin, where sometimes the output is heads and sometimes tails. (To be sure, by importing further, perhaps unknown, input factors, these many-valued examples can be converted into plain functions.)

Secondly, connected with the first, kategory composition is always assumed associative, i.e. where the requisite compositions are defined,

Asc. $(f.g).h = f.(g.h).$

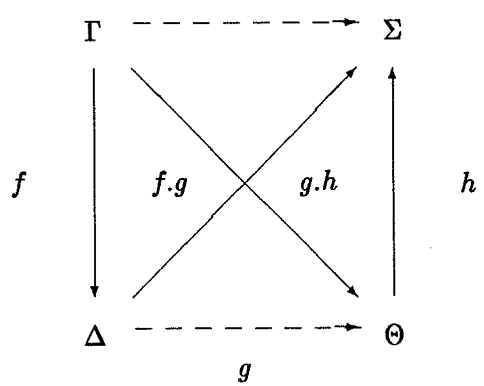
This equation is commonly represented pictorially, in the somewhat misleading claim that the following diagram is commutative (with direct conventions for function compounding adopted):

[8]Here = is so far an identity determinate of the background epitheory or, in Curry's further terminology, of the U-language.

[9]That is, mathematical theory introduced by Eilenberg and MacLane, not the traditional theory extending from Aristotle through Kant to recent significance theory. As the mathematicians' choice of terminology was unfortunate, further overloading an already ambiguous term, the liberty of varying it is straightaway taken. For *category* or *categorial* theory, for instance, is at least three ways ambiguous: between linguistic, mathematical and philosophical streams.

[10]See for one example, MacLane, p.1.

$$(f.g).h = f.(g.h)$$



There is no reason however why this indifference to association of order should hold for processes or procedures in general. As it happens, intuitive counterexamples abound. Chemical and biological processes are often decidedly sensitive to association. Consider a practical case with enzyme glues. The multi-step process of mixing the glue components and then applying them sometimes works, i.e. succeeds in bonding appropriate objects, but the process of applying one component to the objects and then the other never works. It is the same with childrens' glue, involving, in this order, the processes of adding water, adding flour and mixing, applying to paper. A more sophisticated example from the edges of applied chemistry, cooking, concerns the process of making Spanish coffee. The usual steps are: add coffee, then alcohol, then add cream. But if the cream and alcohol are associated first, the cream will curdle. The basic genetic process of combining male and female cells in a suitable chamber depends heavily for its success on correct association, as do many other parts of biogenetics. A different style of example comes from linguistics. Consider language functionally (as with λ-categorial grammars) and look at the association of adjectives e.g. the differences in different associations of strings like 'fat little old bum'. Or most simply, consider elementary mathematical functions construed as processes. The processes of addition and multiplication, for instance, are not associative; for many a and b, $(a+b)\times c \neq a+(b\times c)$, e.g. $(3+4)\times 5 \neq 3+(4\times 5)$.

The intended generalisation of the notion of kategory (itself really a certain structure of functions) is accordingly a *faktory*, a system of processes (including "identity" processes) defined on structures. In the usual set-theoretic jargon, a faktory is represented by a system $\langle K, R, \circ \rangle$, where K is a set of structures (or worlds), R is a set of processes (including identity ones) on $K \times K$, i.e. for f in R such that $\Gamma \rightarrowtail^{f} \Delta, \Gamma$ and Δ are in K, and $\circ$ is a composition operation on (interlocking) pairs of processes. In some formalisations of the notion of kategory, functions supplying domains and codomains (or ranges) of functions in R are included in the system specification. In part this is due to a questionable endeavour to avoid set-theoretic foundations for mathematics; in part it reflects an attempt to be complete (though the usual formalisation is conspicuously incomplete). Much as in relation theory, where f is a procedure from Γ to Δ, then Γ is the domain of f and Δ the codomain (i.e. in set-theoretic formalism, *Dom* '$f = \{x \epsilon \Gamma : (Py \epsilon \Delta) x f y\}$).

The requirements for identity processes are the same as those for kategories. For every Δ in K, there is an identity i_Δ in K satisfying Id. An "identity" process is, more picturesquely, a Do-Nothing (or vacuous) process: it leaves the structure it proceeds from or applies to exactly as it was.

Now to specify the generalisation: A (*general*) *kategory* is a faktory which satisfies the associative condition Asc generally. A *standard kategory* is a kategory in which every process is a (single valued) function.

PROPOSITION. *There are general kategories which are not standard kategories.*

ARGUMENT. Consider a two structure faktory M with exclusive structures Δ_1 and Δ_2 and just one non-vacuous process f which is not single valued, i.e. in obvious symbols both $x f y_1$ and $x f y_2$ hold, for $x \epsilon \Delta_1$ and $y_1, y_2 \epsilon \Delta_2$. Then M is a general kategory, if a rather trivial one. For, by inspection Asc holds, almost vacuously; the only cases are guaranteed by Id.

PROPOSITION. Asc *guarantees single-valuedness.*

ARGUMENT. Suppose some $f : \Delta_1 \rightarrowtail \Delta_2$ is not single-valued, so, for $x \epsilon \Delta_1, x f y_1$ and $x f_2$ with $y_1 \neq y_1$ and $y_1, y_2 \epsilon \Delta_2$. Let $i = i_\Delta$ and consider the results of $(f \circ i) \circ i)$ and $f \circ (i \circ i)$ applied to input x. If f yields in one association y_1 and in the other y_2 then, by Asc, $y_1 = y_2$ which is impossible. (Strictly the notion of identity becomes problematic here for multiple-valued "functions").

The mathematical way forward is initially clear. Generalise what in standard category theory can be appropriately generalised. The practice of generalisation initiated here is exceedingly common in mathematics. Once a theory, such as group theory or lattice theory or some such, begins to get worked out, generalisations are made, to semi-group theory and the like, in cases where interesting mathematical properties survive or emerge. So it is with mathematical category theory, where natural generalisations in the shape of process and procedure theories are already ripe for mathematical processing: whence semi-category theory. The comparison with semi-group theory is particularly apposite, because semi-group theory results from group theory by a similar generalisation, namely abandonment of associativity. That mathematical way is not however a direction we shall attempt (under instructions from editors not to veer into mathematics). Let us instead descend, jumping down from the rarefied general theory to logico-philosophical nitty-gritty, firstly to action action.

3. Relevant action logics and immersing action theory

Action logics have been shaped by the routes to them through action.

On routes to action theory, and the dominant Humean theory

There are various strikingly different routes into the action enterprise. Philosophical routes, which characteristically lead to what is called "action theory" or "philosophy of action", and which thereby disclose main logical functors, divide into three broad categories:

- from philosophy of mind, where a major issue is the relation between mental states and connected action. Hardly necessary to add, much interest is concentrated, particularly where Anglo-American philosophy has an impact, upon reduction of some sort: reduction of distinctively mental attributes involved in (deliberative) action — such as intention, desire, belief, thought, deliberation — to "acceptable" nonmental matters. These reduction bases include as well as more old-fashioned and biological behaviour and structures, much new-fangled stuff like neural nets, computer software and programs, and avant-guard linguistic structures.

- from philosophy of language, where action discourse, involving much the same functors as before, and associated semantics for them, are both problematic. Again major difficulties are produced by reductive philosophies, which seek to avoid ontic commitments to objects of action and action states (or else to minimize upon their commitments, whence fruitless disputes as to numbers and identity of actions in single processes. Such disputes derive from erroneous ontological assumptions: see JB).

- from ethics and philosophy of law.

> In ethics, most of all, questions about action arise on every front. Responsibility and excuses cannot be adequately discussed without an analysis of ability and inability, and an account of the difference between intentional and unintentional acts. Ethical theories, such as utilitarianism, cannot be properly assessed without an understanding of the relationships between acts, consequences, circumstances and motives (Goldman p. v).

Similarly in the theory of law, of torts especially, questions about action arise on many fronts: among others, identity of actions, agency responsibility for action, right to act. Seminal Scandinavian work on logics of action arose directly from issues in ethics and law. Von Wright turned to a logic of action to surmount difficulties in deontic logic. Kanger was led to a logic of action in providing an analysis of legal theories of rights (for fuller details see Segerberg 89, p. 237 ff.).

It is remarkable, given the richness of routes to action theory, that investigators such as Åqvist should declare that action theory is the outcome of unsuccessful attempts to mitigate the paradoxes and problems of (dyadic) deontic logic (an unconvincing excuse is that he is trying to give a survey of deontic logical theory). Moreover, although Åqvist asserts that the action logic 'movement is highly interesting and promising for the future', he says nothing at all about how it helps (see p. 664). But something of the intended role can be glimpsed. For according to Åqvist, to reach viable deontic logics, 'such logics ought to be combined with a *logic of action*' (p. 663). In support of this he appeals to van Eck's diagnosis: 'the languages of the current systems of deontic logic are far too poor to function as a satisfactory medium for formulating cues for the moral agent'. But, to the contrary, it is not *just* the poverty of the languages, but, more important for deontic theory, the *inadequacy* of the logics (see MD). Therewith the tenuous linkage to action theory as a rescue package is snapped.

Once appropriately motivated from elsewhere, action theory proceeded to turn into its own enterprise. It has assumed two dominant, hitherto only loosely related forms, according to whether it is pursued by analytic philosophers (mostly American) or modal logicians (usually Scandinavian linked). Dominant philosophical theory is centred upon a desire-belief model of action, which, in the quest to establish a worthy tradition, is honorifically ascribed to Hume (as to the already remarked questionability of the ascription, see further MacIntyre pp. 339–340). Dominant logical theory is much leaner, so far focussing upon what is only part of Humean theory, upon some propositional form of *doing*. Exactly how the so-called Humean action theory is made out remains obscure. But a central inference takes something like the following form:

H. x desires that C
x believes that B will produce C
x does "B", e.g. x brings it about that B
all going well, C.

The main difficulties lie not with the regularly presented desire-belief-cause premisses, but with what is taken to be inferred from them, how action derives through this sort of "practical" "syllogism". For there are a host of reasons why x may not do "B" (i.e. decoded, what makes it the case that B): timidity, fear, weakness of will, incapacity, force of circumstances, etc.,etc. A promising way of closing the (first) inferential gap is authentically Humean, namely converting the initial conclusion to a *motivational* claim, such as that x is motivated to do B, or disposed to bring it about that B. Indeed there is a marked tendency among very modern Humeans to make the connection *two* way, and to close the inferential link by construing the resulting connection as analytic (if it is not analytic on motivation, then it is on *rational* motivation, so it is imagined). Thus:

J. x is motivated to do B iff, for some C, x desires C and x believes B will cause C.

But this biconditional should be scrapped, because the new half entirely lacks plausibility (except under crude psychological reductions of values and the like). For example, x may be motivated to do B not because x desires C but because, little as x desires C, C is meritorious, or will benefit some deserving creatures (whom x may personally detest) or similar. Similar objects apply to the Humean theme that creatures, or humans anyway, are always motivated by desire as guided by belief (concerning causation).

In principle, opinions about the Humean desire-belief modelling trifurcate. As well as Humeans there are

• sub-Humeans. According to this recently fashionable stance, the Humean modelling is extravagant, containing both desires and beliefs when *one* is enough. Under the main reduction pressed, it is claimed desire can be removed, because belief alone can effect the whole task. Desire, it is said, is but a species of belief, for instance, belief as to what would be good.[11] But evidently such an unlikely proposal does not escape from the general orbit

[11] Such themes are both presented and criticized in a series of articles (by Price, Lewis, and others) appearing in the English journal *Mind* from 1988 on.

of what cruder Humeans would count under desire and preference, namely matters of what would be desirable, preferable, good. (Decision-theoretic variations on the Humean modelling are of course premissed upon subjective probabilities and desirabilities.)

• super-Humeans. According to this stance, adopted here, more is needed for an adequate modelling of action, its motivation and explanation, than the impoverished Humean modelling incorporates. The more includes, as well as what is adduced below, other schemes, which include activating factors such as recognised values and reasons (what Hume himself did not altogether neglect) and also a range of acknowledged retarding factors.

But formalizing even as much as is supplied by Humean modellings already presents a serious challenge. For the central inference, H, however problematic, exceeds the formalization capabilities of standard modal-based action logics. Motivational complications and other variations simply make matters worse. Standard logics often include only a functor D (from *Does*) corresponding to process accreditation and typically construed in such stronger terms as *brings it about that, makes it the case that, sees to it that, realises that*, or similar.[12] That much, which naturally does not exclude more, appears logically central and essential. Åqvist informs us that '... the distinguishing mark of a logic of action [is] the presence in its basic language of a special "casual" operator of *agency* expressing that an agent *brings it about, sees to it, makes it true* that so-and-so is the case' (p. 663). Much of the stock Scandinavian literature seems to concur. Segerberg in his recent survey keeps returning to *does*, which he renders functorially: the agent sees to it that (89 cf. p.23–4). Kanger, though he sets out with *causes that* replaces it with *sees to it that* (also linked to the imperatival *do*!). And so on.

An adequate action logic, properly placed to capture formally inferences of the dominant philosophical theory, has to be substantially richer. A fuller action logic has to be a multiple functor logic, including as well as D_x, such functors (for each requisite subject x) as

W_x: *x desires that, x wants* (it to be the case) *that*

B_x: *x believes that* (*x takes it that it is probable that*)

$\supset$ for causal implication: *that ... causes* (*it to be the case that*) ....

With this symbolism we can represent the first critical part of H (given perhaps a tense shift) as follows:

[12] Pörn actually contends, for reasons he does not divulge, that an improved reading for D_x, over 'x brings it about that', is 'it is necessary for something which x does that'! However it does make the (relative necessity) style of the logic for D plainer, and allows for a free substitution of "identicals".

H_s. $\mathsf{W}_x C, \mathsf{B}_x(B \supset C)/\mathsf{D}_x B$.

To encompass the remainder of the original inference, H, and recommended improvements, some further functors, which appear to have gained no logical exposure, are needed: propositional analogues of 'x is motivated to', henceforth M_x, and 'all going well (according to schedule, etc.)', henceforth P. Then the improved conclusion to H_s is $\mathsf{M}_x \mathsf{D}_x B$. That result does not however justify $\mathsf{P}B$, for instance because x may not do B though motivated to so. Nor is the originally intended inference

$$\mathsf{B}_x(B \supset C), \mathsf{D}_x B/\mathsf{P}C$$

tight, because x's beliefs about causes may be astray. The functor P, which we shall not investigate further here, behaves logically rather like the functors "Plausibly" (also "arguably") or "It is plausible that", whence the choice of symbolism. The motivational functor, M, appears to operate like a dilute intentional functor, and accordingly will be picked up again after intention is introduced below.

Most action logics do not include so much apparatus, many play around with little more than principles and details for D. That is not an unimportant matter. But it does mean that such action logics are not merely ill-equipped, but unequipped, to deal with central issues in action theory, such as received explanations of action. We shall bring in *much* more apparatus, piece by piece, beginning, as now customary, with D. Nor is it as if a rich logical apparatus is being wheeled on to the philosophical logical stage for the first time. Virtually all the rest of the apparatus has been extensively used elsewhere, often informally, especially in epistemology and philosophy of science, so it needs no detailed new introduction. Belief and desire functors come together also in decision theory. And so on.

What has been learned, in some regions, from extensive experience with these functors and like functors, is this: that standard analyses, which are modal, are seriously inadequate in multiple respects (the reasons, many of which are well-known, are summarised in PLI). Some practitioners have recognised some of these troubles, and are beginning to adjust their theories accordingly (e.g. von Wright, da Costa, Fuhrmann). Naturally the same inadequacies reappear with, and within, action theory.

Very many of these troubles can be straightforwardly avoided by the simple procedure of shifting to a relevant base logic. Conveniently too, many of the functors we need to consider have already been investigated in relevant settings: *belief, desire, cause* ...(see PLI and RCR). The settings are like those for modal logics, semantically relational structures, but the relevant structures are based on a more generous complement of situations. What has not been considered before in a relevant setting is the functor D itself, taken to typify action in action logic; likewise several functors in its vicinity, such as those of *trying, intending, sustaining*, and so on.

On the relevant logic of doing, D

Where A is a wff, so now is $\mathsf{D}_x A$, basically decoded as "Credit x that A (but often construable in context as "x makes it the case that", "x brings it about that A", or similar) and paraphrased as "x *does* A". In the usual fashion subscripts $x, y \ldots$ for agents are omitted where context allows. What is of immediate interest are logical (and semantical) properties of D. Since action is certainly conjunctive, i.e.

C. $\quad \mathsf{D}A \mathbin{\&} \mathsf{D}B \rightarrow \mathsf{D}(A \mathbin{\&} B)$,

making it true that A and making it true that B entails making it true that A and B, it is tempting to invoke the following distribution rule:

RC. $\quad A \rightarrow B / \mathsf{D}A \rightarrow \mathsf{D}B$.

That is, where A implies B is a theorem, so is $\mathsf{D}A$ implies $\mathsf{D}B$ (to state the distribution of D in a minimal form; there are stronger inferential readings). It is tempting because these two principles render D a systematic connective (as explained and analysed in PLI). That carries the big advantage, other things being in order, of enabling a normal semantical rule, analogous to that for necessity $\Box$, to be adopted. That is, $\mathsf{D}A$ holds at situation a iff for every situation b such that $R(\mathsf{D})ab$, i.e. D-accessible from a, A holds at b; in seventies symbols, $I(\mathsf{D}A, a) = 1$ iff $(b).R(\mathsf{D})ab \supset I(A, b) = 1$. The situations of $K = \{a, b, c, \ldots\}$ of such semantics are any sorts of items; they have no particular structure, only accessibility interrelations. Adequacy of the semantics is demonstrated as usual (see PLI).

What is perceived to be a problem with even this much structure, with RC applied across a standard implicational linkage, is the following principle

DAdd. $\quad \mathsf{D}A \rightarrow \mathsf{D}(A \vee B)$,

which results from application of RC to Addition: $A \rightarrow .A \vee B$. The principle, sometimes called *Ross's paradox* (for D), has been given much prominence in European philosophical logic, where its undesirability tends to be repeated uncritically from author to author. In an action context, what is assumed paradoxical is, for example, that seeing to it that a letter is mailed should entail seeing to it that it is mailed or it is burnt. A proper worry, which had however been improperly transferred to this principle, is that seeing to it that a letter is mailed or is burnt can be satisfied by burning it, whereas seeing that it is mailed is certainly not satisfied in this fashion. (A postman's job would become refreshingly easy; he performs his task by disposing of the mail in the nearest dump, such as a river in Rome.) But satisfaction of action requirements does not normally operate in this converse implication fashion. A comparison should help make the point. While the converse of C,

M. $\mathsf{D}(A \,\&\, B) \rightarrow .\mathsf{D}A \,\&\, \mathsf{D}B$,

of course follows, like DAdd, upon applying RC, it is easy, or easier, to see that what satisfies $\mathsf{D}A$ may be far from satisfying $\mathsf{D}(A \,\&\, B)$.

The idea that satisfaction or implementation does somehow transfer in the case of disjunction, and that there is something problematic (at least) about some examples of $\mathsf{D}A$, appears to turn upon an application of

DSyll. $(A \vee B) \,\&\, \sim A \rightarrow .B$;

that $\mathsf{D}(A \vee B)$ when $\sim A$ is seen to (or even when A is not seen to) forces $\mathsf{D}B$. But while modally $\mathsf{D}(A \vee B) \,\&\mathsf{D} \sim A \rightarrow .\mathsf{D}B$, using RC, such an implication is relevantly inadmissible, because DSyll is. Thus, duly illuminated, Ross's paradox is a modal paradox, allied to standard modal paradoxes like (what DAdd thereupon yields from DDSyll) the form $\mathsf{D}A \,\&\, \mathsf{D} \sim A \rightarrow .\mathsf{D}B$. And it is relevantly removed with them. (For elaboration of this approach in the related setting of modal logic, see MD). Without wanting to associate with the thriving business of making paradoxes respectable — classical logicians, latter-day sophists, are already sufficiently skilled at this — there is a distinction to be drawn between genuine and bogus paradoxes. Addition, $A \rightarrow .A \vee B$, which does fail for (Kantian) containment, holds for implication and is not a paradox therefore, by contrast with $A \rightarrow . \sim B \vee B$, which is not bogus. Similarly $\mathsf{D}A \rightarrow \mathsf{D}(A \vee B)$ and Ross's examples present but bogus paradoxes for *relevant* action logic, by contrast with $\mathsf{D}A \rightarrow \mathsf{D}(\sim B \vee B), \mathsf{D}(B \vee \sim B)$, etc., which are genuinely paradoxical.

Nonetheless, a relevant action logic without principle DAdd can be readily supplied, in either of two ways:

1. Modifying RC, reining the premiss back to a coimplication (in effect, a coentailment). In a relevant setting that strategy appears unnecessary

— because, as already seen, the systematic distribution rule does not induce modal problems — and undesirable — because expected outcomes such as M are sacrified and have to be separately postulated. Moreover RC is recoverable given M, as the following sketch argument shows:

$$A \to B/A \leftrightarrow A \,\&\, B/\mathsf{D}A \to \mathsf{D}(A \,\&\, B)/\mathsf{D}A \to \mathsf{D}B, \text{ using M.}$$

2. Modifying the implication $\to$ to a relevant containment connection for which Addition does not hold. While such a strategy has much to recommend it for functors which depend on content, such as those for belief and assertion, it does not for more translucent functors such as those for obligation and action, which distribute over implication.

There is more to a relevant logic of action than RC and C. Because D is a success functor, because an agent does not make it true that A without A, the principle

T. $\mathsf{D}A \to A$

must hold. Because of T the coupled process-achievement features, of *bringing about*, which von Wright suggests separating, are integrated; such a separation is in any case artificial since any action product is essentially coupled to a process (cf. von Wright himself 83 pp. 107–8). T also has corollaries for mixed agent principles; for example it follows $\mathsf{D}_x\mathsf{D}_yA \to \mathsf{D}_xA$.

From T it follows, by implication logic,

Con. $\mathsf{D}A \to \sim \mathsf{D} \sim A$,

a qualified consistency principle. Certainly it similarly follows, by virtue of Non-contradiction, $\sim (A \,\&\, \sim A)$,

Ñ. $\sim \mathsf{D}(A \,\&\, \sim A)$,

no agent sees to it that an explicit contradiction obtains. The normal modal inverses of such principles, as for instance

N. $\mathsf{D}(A \vee \sim A)$,

do not hold but are rejected. Naturally their negations are not asserted, for reasons soon to be advanced. The principles themselves do not ensure in the normal way because the (necessitation) rule

RN. $A/\mathsf{D}A$

certainly fails. Fortunately, in a relevant setting, it naturally fails.

It will be evident that rejections are as important as assertions in fuller characterisations of relevant actions logics. Rejections, such as $\dashv \mathsf{D}(A \vee \sim A)$ and $\dashv\sim \mathsf{D}(A \vee \sim A)$, can of course be taken up as assertions in richer logic with apparatus like that of assertional quantification, as $(Pp) \sim \mathsf{D}(p \vee \sim p)$ and $(Pp)\mathsf{D}(p \vee \sim p)$ respectively.

Relevant action logic *automatically* delivers certain features that von Wright regards as meritorious properties in his later action logic (the second of those logics developed in 83 p. 168ff.): namely that the logic is 'intensional and not extensional' (p. 183). For, says von Wright, 'consider $\mathsf{D}p$ and $\mathsf{D}(p \,\&\, (q \vee \sim q))$ or $\mathsf{D}(p \,\&\, q \vee p \,\&\, \sim q)$. The last two are equivalent, but the first is not equivalent with either of them' (p. 182 functors rewritten). Exactly the relevant situation, inasmuch as $\mathsf{D}(p \,\&\, (q \vee \sim q)) \leftrightarrow \mathsf{D}(p \,\&\, q \vee .p \,\&\, \sim q)$, but $\mathsf{D}p \not\leftrightarrow \mathsf{D}(p \,\&\, (q \vee \sim q))$. However these results emerge for utterly different reasons. Under relevant logic these results result because $p \,\&\, (q \vee \sim q) \leftrightarrow .p \,\&\, q \vee .p \,\&\, \sim q$, whereas $p \not\leftrightarrow p \,\&\, (q \vee \sim q)$. However, according to von Wright, ' "$p \,\&\, (q \vee \sim q)$" describe(s) the *same* state of affairs as "p". ... "$q \vee \sim q$" may be said not to describe any state of affairs at all. The identity of actions (as opposed to states of affairs) is in a characteristic sense "sensitive" to their descriptions' (p. 183). Conventional defective classical wisdom? Not really: the D-equivalence fails because on von Wright's (distribution) principles, in diametrical contrast to usual modal action logics, 'the component $\mathsf{D}(q \vee \sim q)$ is self-contradictory' (p. 183). Von Wright offers no justification — other than what it does not have, obviousness — for this result, and it seems wrong. For imagine an intuitionistic mathematician finely tuning a subtheory to make sure that the law of excluded middle holds good in certain critical cases.

On the relevant logic of trying or striving, T

A functor that has long played an active part in action theory and psychology is that of *trying*, or in more vigorous form, *striving* (*for*). Fundamental in Spinoza's rational psychology was the notion of *conatus*, (innate) endeavour or striving — from the Latin infinitive *conari, to try or attempt*. According to Spinoza, all living things are animated by *conatus*, the endeavour, or desire to perpetuate their being. The principal striving is indeed desire or appetite; happiness is equated with desire satisfied (or realised) and sadness with desire not satisfied. Then all other emotions are defined in terms of this triple: *desire, happiness* and *sadness*, with the help of a few notions from metaphysics, notably causation. Exact action theory, which has already enlisted such functors as those of *desire* and *realisation* is almost poised to reassess Spinoza's rational psychology. In late nineteenth century German philosophy, the role played by *conatus* (and by desire in Hobbes, desire or will being the final link the chain of appetites leading to action) was assumed by *will*, later *drive*, a principal element in decision. In Nietzsche the drive to power became the fundamental motive. This intellectual history influenced Freudian and depth psychology which assigned a central role to *conatus*, often rendered as drive.

More recently Fitch made *striving* basic in his embryonic theory, the first explicitly relevant theory of action. The theory is in effect based on a functor of trying T. The expression $\mathsf{T}_x A$, read unidiomatically: x strives for A, expands to : x strives to make it the case that A, or: x strives to do (or realize) A. In Fitch we encounter a less ambitious repetition of part Spinoza's program, along with independent developments (such as explicit logical limitations upon what agents can accomplish). For Fitch, like Spinoza before him, proposes to define an astonishing range on intentional notions in terms of *striving*, again in combination with partial *causation*: doing, knowing, ability, desiring, valuing (p. 191).[13]

As adapted to the present framework, the axiomatisation of T that Fitch proposes is a variation upon the following:

$\mathsf{T}(A \,\&\, B) \leftrightarrow .\mathsf{T}A \,\&\, \mathsf{T}B$ (conjunction elimination and introduction, p. 137)

$A \rightarrow B / \mathsf{T}A \rightarrow \mathsf{T}B$ (T distribution).

In fact Fitch works with propositional identity, characterised exactly like first degree entailment, in combination with strict implication. As entailment is said to be defined in terms of propositional identity, as $A \rightarrow B$ iff $A \,\&\, B = A$, and identity amounts to two-way entailment, i.e. coentailment, it is evident that propositional identity also amounts to coentailment. Fitch's first axiom scheme for identity yields the replacement principle $A \leftrightarrow B / C(A) \leftrightarrow C(B)$ and is effectively a strict strengthening of that rule. Remaining coentailment axiom are just those of first degree coentailment (p. 140). Now the

[13]It is sometimes supposed that intending will similarly succumb to some such account as the following: $\mathsf{I}_x A$ iff $\mathsf{T}_x \mathsf{D}_x A$, i.e. in effect x intends to do ψ iff x strives to bring about ψ. But deficiencies are evident. Agent x may be weak-willed and sometimes not act upon good intentions. The converse also has problematic features if the agent is insufficiently cognisant of what it is about. Functor I of intending is treated in the next section.

replacement principle has as a special case (the full inductive case for T)

$$A \leftrightarrow B / \mathsf{T}A \leftrightarrow \mathsf{T}B$$

which yields and is tantamount to T distribution. For

$$A \to B \leftrightarrow .A \,\&\, B \to A / \mathsf{T}(A \,\&\, B) \leftrightarrow \mathsf{T}A$$
$$/(\mathsf{T}A \,\&\, \mathsf{T}B \leftrightarrow .\mathsf{T}A) \leftrightarrow .\mathsf{T}A \to \mathsf{T}B,$$

using the definition of $\to$ and the first scheme for T. Whence distribution. Since the definition of $\to$ in terms of $\leftrightarrow$ or $=$ does not succeed for higher degree relevant logic (see FL), it is preferable to settle directly for T distribution.

Now, in terms of T, it looks straightforward to recover the basic relevant logic of doing D. For what is doing but trying to do and succeeding? *If* so, D can be approximated thus: $\mathsf{D}A =_{Df} \mathsf{T}A \,\&\, A$. Then axiom scheme T is immediate from the definition.

*ad*C. $\mathsf{D}A \,\&\, \mathsf{D}B \leftrightarrow (\mathsf{T}A \,\&\, A) \,\&\, (\mathsf{T}B \,\&\, B)$
$\leftrightarrow \mathsf{T}(A \,\&\, B) \,\&\, (A \,\&\, B)$
$\leftrightarrow \mathsf{D}(A \,\&\, B)$

*ad*RC. Now $A \to B / \mathsf{T}A \to \mathsf{T}B$. Also as $A \to B / A \to B$,
$A \to B / \mathsf{T}A \,\&\, A \to \mathsf{T}B \,\&\, B$, factoring in $A \to B$ as relevant rules permit. Whence RC for D.

It might be objected against such a definition that an agent may do something without (really) trying. (The 'really' is something of a give-away.) But if there is no trying at all, and the outcome happens by accident or coincidentally, then surely in an obvious sense (an obviously stronger sense) it is not done *by* the agent? Such an objection, which looks more effective against *striving* than *trying*, would also tell against Fitch's more elaborate definition of *doing*. According to Fitch,

> *D*1. (does p) $\equiv \exists q$ (strives for $[p \,\&\, q]$ & (strives for $[p \,\&\, q])Cp)$).
> This means that an agent does p if and only if there is some (possible or impossible) situation q such that the agent strives for p and q, and a result of this striving is that p takes place (pp. 140–141).[14]

[14]Here C is partial causation, which can be defined in term of relevant causal implication (of RCR): ACB iff $(PD).A \,\&\, D \supset B$. Note that in making room here and elsewhere for impossible situations, Fitch, who claims to anticipate first degree entailment, has also glimpsed an important feature of the semantics.

Part of the simple approximation, when DA then TA & A, follows from this elaborate form, but not all the remainder. But, interestingly, the axiomatisation of the basic logic of D does emerge given expected properties of striving (i.e. T-like properties). Unfortunately, not only is the simple approximation too simple, so is Fitch's curious proposal. Neither makes due allowance for unintentional and unstrove for doings, as when a shopper's trolley bumps a pyramid of fragile produce in a super-market. (Fitch could have avoided *this* difficulty by removing p from what is striven for, leaving that to some more adequately connected q.)

In his *Beginnings* (i.e. 89), an important (if incomplete) survey of logics of action, Segerberg proposes a modal reconstruction of Fitch's theory. This modal reconstruction is of independent interest, as it includes some movement in a relevant direction (in terms of restrictions that wff in certain postulates share no propositional letters), movement not encountered in Fitch at all. In fact Segerberg's interesting reconstruction remains removed from Fitch both in intention and in results. For example, Fitch is at pains to point out that his (already relevant) system does 'not have such theorems as $p = (p \;\&\; [q \vee \sim q])$' (p. 140), and accordingly does not permit intersubstitution of such forms within action functors such as T. By contrast, Segerberg's reconstruction does allow problematic substitutions. A relevant reconstruction of Fitch, Spinoza, and contemporary *conatus* theory generally, remains to be accomplished.

On the relevant logic of intention, I

Functors that play a large role in action theory are those in the vicinity of intention: not merely intending itself, but wanting, desiring and deliberating. Indeed these functors tend to play a grander role than action theory really warrants, because of the mistaken assumption that all (normal) action is intentional, along with the mistaken conflation of intending with wanting or (when that is removed) the mistaken location of intention within the want-belief nexus. Defective accounts of intentionality abound, partly because there is heavy pressure to reduce it to something else. Though it has a logic rather like belief, the notion is very different, with a quite different range of application. Nor is an agent's intention tied to what it wants, or wanted, to do, because it may have to perform (intentionally) other actions it does not want to do, or have done, in order to achieve what it does want or values (cf. Goldman, p. 130). Nor is an action intentional when it realises approximately, or indirectly, what is wanted or sought, because, even though directed, things may still go wildly astray: successful direction is something more. And so on, through subtler and more oblique combinations of wants and beliefs. Want-belief reduction is, once again, not what is wanted: intention needs independent investigation.

Let us introduce the function I, construed awkwardly, in order to stay within the propositional setting, as: the given agent intends that, or: that ...is intended by the agent. Thus I_x reads: (*agent*) *x intends that*. A certain familiar amount of twisting is needed in order to push variant forms into *that* forms; for instance "x intends to do ψ" becomes "x intends that x do ψ". Then a preliminary analysis of "x does A intentionally", and likewise it is sometime imagined of "x does A deliberately", takes the conjunctive form: $\mathsf{D}_x A$ & $\mathsf{I}_x A$. (But as is well known, from Montague and Gettier, conjunctive analysis of modifiers and intensional conjuncts is liable to produce paradoxical or bizarre outcomes. For instance, it is a joke that large mice are large, that motivated doings are done.) In fact deliberate action is separate from intentional action, for all that their equation is sometimes suggested (e.g. Segerberg looks as if he is equating deliberation with forming an intention in 82 p. 23). Plainly an agent may deliberate — consider alternatives and weight up considerations mentally — without reaching a result or forming an intention. If an agent takes action deliberately, for instance because some time-line expires, then, though intentionality in the weak sense of mental activity may be met, the agent may nonetheless have formed no clear intention, no design, not be set or bent on an object. The breakdown of the converse is clearer still. An agent may form an intention, immediately, without deliberation; no consideration, no pondering, no requisite processing. Deliberation demands, then further investigation, beyond that we intend to direct towards intentionality.

While a logic for I is meagre, it is not null. Thus the idea of disposing of intentions as a bundle of more or less arbitrary propositions (mere *that* ps, on Segerberg's more recent account) can be dismissed. But certainly some of the logic that has been laid on intention is a bit thick. For instance, Segerberg has variously supposed that an agent always has some intention (p. 78); indeed just one intention, i.e. in effect $\mathsf{I}_\alpha A \;\&\; \mathsf{I}_\alpha B \rightarrow .A = B$. But an agent can have a bundle of intentions, more than one of which may be operational in propositional settings. Further, an agent can act, for example on the spur of the moment, or in response to an immediate issue, without always drawing upon a ready-made intention set or forming a further intention. But logic there is. The propositional logic of I does include a full range of propositional transformations, for instance $\mathsf{I}(A \;\&\; B) \leftrightarrow \mathsf{I}(B \;\&\; A)$, $\mathsf{I}(A \;\&\; A) \leftrightarrow \mathsf{I}(A)$, $\mathsf{I}(A \;\&\; (B \;\&\; C)) \leftrightarrow \mathsf{I}((A \;\&\; B) \;\&\; C)$, $\mathsf{I}(A \;\&\; (B \vee C)) \leftrightarrow \mathsf{I}(A \;\&\; B \vee .A \;\&\; C)$, etc. What ensures all these? Even if $\leftrightarrow$-replacement fails, as it may for stronger implication (cf. PLI), $=$-replacement does not. That is, the rule,

$A = B, D(A)/D(B)$, with I in its scope, will provide such combinatorial results.

As observed the logic of intention resembles, at a propositional level, that of belief, and similar trouble spots are encountered. But of course the logics are different, evidently when other functors are brought in; for instance knowledge implies belief, while knowledge does not imply intention. Trouble spots include principles yielded by first degree $\rightarrow$ transmission, such as $\mathsf{I}(A \,\&\, B) \rightarrow .\mathsf{I}A \,\&\, \mathsf{I}B$ and $\mathsf{I}A \rightarrow \mathsf{I}(A \vee B)$ and its mate, and also the converse of the first, $\mathsf{I}A \,\&\, \mathsf{I}B \rightarrow \mathsf{I}(A \,\&\, B)$, adjunction. But the supposed trouble with the first, intentional simplification comes from rendering the conjunction intensionally, as if the components were related. Thus it is claimed, correctly, that an agent may intend A and B in concert without intending each separately. Only then the claim should be symbolized differently, for instance through such intensional composition propositions as $\mathsf{I}(A \circ B) \rightarrow \mathsf{I}A$, which get duly rejected. The objections to the second principle and its mate are different. They include such observations as that an agent who intends A may find B irrelevant or may even lack the information or conceptual apparatus B presupposes. As such observations themselves suggest, additional baggage like that B brings with it can be shed by using an analytic implication or, in this context, relevant containment logic. Closure of intention under the first-degree implication of such a containment logic will deliver intentional principles like $\mathsf{I}(A \,\&\, B) \rightarrow \mathsf{I}B$, $\mathsf{I}B \rightarrow \mathsf{I} \sim\sim B$ and $\mathsf{I}B \rightarrow \mathsf{I}(B \vee B)$, but not $\mathsf{I}A \rightarrow \mathsf{I}(A \vee B)$. Such a rule we accordingly adopt (semantical details are readily pieced together from RCR and PLI).

Adjunction is open to objection also, though not on all the grounds that afflict addition, since no new propositional content (in the shape of irrelevant B) is introduced. Both A and B are given, and virtually all agents will have a suitable concept of conjunction. Even so an agent may not put intentions together, even though it appears reasonable to do so. If intending involves "having in mind" then adjunction will indeed fail for some agents. This is not so remarkable, and not destructive of logical theory. For certainly adjunction does break down functors, such as assertion and explicit belief, not so remote from intention. A strategy, which handles the business (as explained in PCI) takes adjunction as an optional extra, which holds for certain agents, namely *fully adjunctive* agents. But adjunction is not taken as universally valid.

We are now likely to encounter, but are well prepared for, several puzzles Segerberg has assembled, puzzles which provide (as Russell might have said) a fine test for any logic of intention.[15] Most of these puzzles are in fact

[15]Segerberg works with a functor *Int* which applies to events, but is applied in examples

familiar from other reaches of intensional logic, and indeed met in much the same way through relevant logic in the case of intention as elsewhere.

The first puzzle turns on the impact of incompatible intentions through adjunction and replacements. Oedipus intends A (e.g. that he marry this woman, viz Jocasta) and intends B (e.g. that he avoids marrying his mother), so, as an adjunctive agent, he intends A & B. But A & B is de facto impossible, Oedipus intends to do what is impossible. But then, classically but not relevantly, A & B =F, the false proposition (in Segerberg's terms $A \cap B = \emptyset$, the null set). But Oedipus never intended F ('$\emptyset$ was surely never an intention of Oedipus' p. 250). Relevant logic almostly automatically resolves such puzzle, since the classical equation of distinct impossible propositions with one another and with the False fails.

While Segerberg rightly observes that the puzzle is not so much an objection to adjunction as to replacement, he goes astray in supposing that it can be removed simply by strengthening replacement to a strict (modal) form (see pp. 251–2). For intentions may be directed towards logical impossibilities. For example while Hobbes intended to square the circle some of his contemporaries intended to design a perfect perpectual motion machine; but they did not thereby have the same intentions because both types of intention were impossible. Here, as elsewhere, the modal framework can and must be transcended: possible worlds give way to wider classes of situations.

However Segerberg does not see how to escape the possible worlds straitjacket, as he effectively concedes in response to 'the second objection' (pp. 252–3), which offers almost a paradigmatic example of what relevant theory was intended to accomplish. "It does not seem plausible that if I intend to close the door, then I intend to close the door and to visit Japan or not." So objected a perceptive referee to the following sort of exhibition of irrelevant tack-on that modal-based theories inevitably validate:

$$\mathsf{I}A \leftrightarrow \mathsf{I}(A \mathbin{\&} (B \vee \sim B)) \qquad \text{Junktion.}$$

The result is inescapable in modal theory so long as I is what it appears to be, a propositional function, because modally that-A and that-$(A \mathbin{\&} (B \vee \sim B))$ amount to the same proposition; they take the same values in every possible world (cf. Segerberg pp. 258–3, who remarks that 'the objectionable feature is deeply grounded in the semantics presented ... How to develop an interesting system without this property is not clear to the author'). As

to infinitival clauses (a straightforward transformation, mostly, of *that* clauses). In any case, the event structure, a Boolean algebra, is homomorphic to what is assumed to be a propositional structure.

$A = A \mathbin{\&} (B \vee \sim B)$, Junktion results at once by substitution. But in relevant logic $A \neq A \mathbin{\&} (B \vee \sim B)$ (as Fitch if effect observed, p. 140). For there are incomplete (possible) situations where $(B \vee \sim B)$ does not hold though A does. So Junktion does not follow, and indeed is easily counter-modelled. For "intention situations" are usually far from complete.

The third puzzle is an elaboration of the first and second, that any modal theory 'will be too strong if one has the logic of intentions of real people in mind' (p. 253, amending the context). Prior had made a similar complaint concerning (modal) logics of belief and assertion (see AC). And Segerberg himself leads into the 'third objection' from a discussion of 'the paradox of the logically omniscient subject: in most epistemic logics, an agent who knows one logical truth knows them all' (p. 253). Such a paradox disappears in relevant epistemic logic, along with epistemic analogues of Junktion (e.g. Segerberg's $KA \leftrightarrow K(A \mathbin{\&} (B \vee \sim B))$), because different logical truths are differentiated. In relevant theory there is no need to be so far removed from "real people" (even if *some* real people may severely test the theory through their unreasonableness and logical perversity).

The puzzle, as duly elaborated, appears to come to this: 'there are real life situations that cannot be modelled in even the weakest of the' modal systems considered (p. 253). But it is, so it is assumed, modal theory or nothing, no logic of substance. If the demand to model real life situations and intentions 'is accepted, trying to meet it would lead to a logic of intention so weak as to be void of content' (p. 254). Segerberg proposes to bulldoze through this dilemma — modal logic, paradoxes and all, or no logic — in the received destructive fashion: suppression of natural people and the artifice of highly idealised agents. Whereupon Segerberg feels able to stand modal theory up again (alongside modal game theory and decision theory, which are presumed not to be damaged by analogous dilemmas[16]), and to display his favoured modal system as 'a logic of *rational action*' (p. 254). The verdict should be that this modal industry does not succeed.

For there are rational actors who make relevant discriminations modal systems cannot accommodate, in violation of Junktion for example. The availability of relevant logical alternatives, to which such agents can certainly appeal, shows that the dilemma modal theorists try to erect is a false one. In between the false choices offered are relevant alternatives. A respectable logic can remain after modal idealisations are shed, and with Adjunction, the example Segerberg focusses upon, abandoned as well. Logics of asser-

[16]But of course they are. A further enterprise, analogous to the present one, is relevant decision theory, which enables many puzzles of decision theory to be straightforwardly resolved.

tion, which should be of this very sort, illustrate the matter nicely. For an assertor may well assert B and also assert C, without thereby asserting their conjunction. But such logics of assertion, though no doubt subtle by crude modal standards, are not difficult, nor devoid of content or interest (see AC again). So it is also with relevant logics of intention; they are not nothing, they can serve real people.

Further action-relevant functors, including motivation; practical inference and more

Unlike intention, motivation does not lend itself at all naturally to propositional formulation. However the difficulty can be sliced through by adopting a neologism: transform "x is motivated to ψ" to "x is motivated that $x\,\psi$", i.e. $\mathsf{M}_x\psi x$. Otherwise motivation is extraordinarily like intention. It seems to function like a *diluted* intention. An agent who intends to perform an action is, ipso facto, motivated to do it, whatever misgivings hold. But an agent can be motivated to do something without intending to do it, because one again of a range of intervening factors (fear, timidity, weakness of will,...). So perhaps I_xA iff M_xA & X_xA for some unknown functor X, like *uninhibited as regards, prepared to implement* (inversely $\mathsf{M} = \mathsf{I} - \mathsf{Y}$). In any case intention looks like motivation *plus*, whence $\mathsf{I}_xA \rightarrow \mathsf{M}_xp$ but $\sim (p).\mathsf{M}_xp \rightarrow \mathsf{I}_xp$. Otherwise the basic logic of M appears to mimic that tendered for I.

Other recognized functors emerge, straightforwardly by compounding, from what is already available. The functor D when applied to A takes no account of the previous state as concerns A, whether A did not hold or did. An agent can see to it that A even when A is already the case. In his later action logic, von Wright wants to take account of what was the case before: 'Some agent may have *produced* this state, i.e. changed the contradictory state into the one which obtains, or the agent may have *sustained* it, i.e. prevented it vanishing, from ceasing to obtain' (p. 169, cf. also p. 170). It seems clear that given an appropriate tense functor, a suitable pastness functor P, analogues of von Wright's functors can be approximated as follows:

B signifying *production* (or destruction) thus: $\mathsf{B}A$ iff $\mathsf{D}A$ & $\mathsf{P} \sim A$
S signifying *sustenance* (or suppression) thus: $\mathsf{S}A$ iff $\mathsf{D}A$ & $\mathsf{P}A$.

Logical syntax and semantics for relevant B and S systems may be obtained by combining those for D, already indicated, with a relevant tense logic (as given in PLI; but note that the functor P will differ from the standard sometime-pastness functor P). The logics are further multiply intensional relevant systems. By imposing appropriate (but not so plausible) axioms, and corresponding modelling conditions, von Wright's system (p. 180) can be approximated. (Other functions obtained by compounding, ripe for relevantization, may be found in Pörn.) Insofar as these logics amount to tensing of multiple functorial systems (which usually means just more functors), they foreshadow modern "process logics", themselves to be relevantly adjusted subsequently.

Enterprise by von Wright, Pörn and others on these "action" functors is coupled with investigation of practical inference and practical syllogism, both under the more general rubric "practical reasoning". Practical reason is a much larger and tougher proposition. Fortunately it is not a topic with which we need to meddle much, as it leads off in a different direction from present variational drill: improving, extending and relevantizing already known systematisations. Accordingly we shall not try to get far into it, but merely poke around at the edges.

In order to formulate *practical inferences* logically, richer multiply intensional logics are required. For these inferences come in a variety of forms, using different intensional functors, and typically two or more different functors each. A schematic form simple quasi-syllogistic practical inferences looks like this:

ΦA $\quad$ Φ represents some volition, desire, value, etc.
A is somehow conditional on B
\- - - - - - - -
ΨB $\quad$ Ψ signifies a doing, obligation, requirement, etc.

A simple example is given by the scheme:

$$\mathsf{W}_x A, B \rightarrow A \text{ (e.g. } B \text{ will produce } A) \,/ \mathsf{D}_x B.$$

The guiding Humean inference, H_8 (which amends the minor premiss to $\mathsf{B}_x(B \rightarrow A)$) can then be seen as an elaboration of the simple scheme which both plugs one gap, that the agent may not be aware of the causal linkage, and covers further circumstances, as where the agent thinks there is linkage but there is none.

Such inferences do not supply deductively valid forms; nor are they easily and convincingly converted into such forms. Even so they may represent reasonable inferential practice (within a modern liberal tradition or paradigm), of which an account needs logically to be given. A first step in this direction is to try to insure that we have relevant logical resources for formalising them. That at least we are beginning to do. A second step is to attempt to present side constraints (recorded in the margin, for instance), under which such inferences may be said to enjoy practical validity. For the inference, H, of modern liberalism (in the narrow British form accurately enough depicted by MacIntyre) where the whole value-theoretic apparatus is supposedly reduced to wants, the side conditions will record something like: reductionistic liberalism, no retarding factors.

Even the prospects of formalisation, however, become problematic should we look at what Aristotle originally expected of practical inference. Namely, it should issue in action. In one of Aristotle's examples (discussed by von Wright 83 p. 1), the premiss pair:

All sweet things ought to be tasted. That thing is sweet,

is taken to issue in the following *action*:

tasting the thing immediately.

(Others suppose it yields an imperative: Taste...!) So long as this is construed in type, not token, form, formalisation does not represent an enormous problem within process theory. It simply exceeds the resources of propositional action logic — which does not include action, only reports upon them (such as that an agent does something). The side constraints are effectively those (broached in recent formal ethics) required to convert practical obligation reasonably to action:

That thing ought to be tasted (now)/(Your) tasting it immediately [constraints].

'Aristotle seems not to have had ... (necessary) means to an end ... argument in mind when speaking of practical syllogism', what von Wright suggests is 'the peculiarity' of practical reasoning (83 p. 1). Remarkably von Wright's own *primary* practical inference (83 pp. 2–3) is a sort of inversion of the Humean means-end form. Whereas H_x works from what is believed to require implementation to deliver a desired end, von Wright's inversion considers what impediments should be removed to avoid frustration of that end. In statement form the inference look like this:

I.	x desires that A	$\mathsf{W}_x A$
	unless x sees to it that $B, \sim A$	if $\sim \mathsf{D}_x B, \sim A$
	- - - - - - - - - - - - - - - -	- - - - - - - - - - - - -
	x should see to it that B	$\mathsf{O}\mathsf{D}_x B$

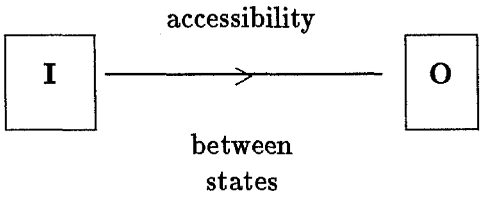
Evidently there is a Humean variant upon this which covers the second premiss by a belief functor B_x. (But there are still divergences of fit, and indeed an exact inversion of the Humean form would be bizarre.)

Once again there has been extensive debate about whether any such forms as I are "practically" valid. Whether any are or not, applied validity can be obtained in this devious sort of way. Suppose for simplicity (only complexity is removed, not generality) that the second premiss is formulated with an implication $\rightarrow$, i.e. it is tantamount to $A \rightarrow \mathsf{D}_x B$. Now define an implicational closure W^c of W as the implicational ancestral of W. So, if $\mathsf{W}A$ and $A \rightarrow B$, then $\mathsf{W}^c B$. W^c is part of what a *rationalised* desire functor would be. Finally introduce a certain personalised *should* functor O, such that whenever W^c then normally O. Then $\mathsf{W}_x A, A \rightarrow B / \mathsf{W}^c_x B / \mathsf{O} B$, whence $\mathsf{W}_x A, A \rightarrow \mathsf{D}_x B / \mathsf{O}\mathsf{D}_x B$. Wonderful. However it is too like trying to patch inductive inference by uniformity premisses. An analogous patching procedure looks unconvincing for direct Humean forms. And side conditions are still needed (e.g. to get from W^c to O).

It is past time these logical chestnuts were cracked: through *nondeductive* inference forms issuing in action relevant conclusions, with warranting (or defeating) side constraints indicated. Investigation of such inferences is no doubt *one* important direction relevant action logic needs to take, to pull action logic and theory together. For action logics do not yet get to grips decently with action theory as outlined; they miss much of the action action. Meanwhile, there is more relevant variational drill to be accomplished, relevantly varying modal efforts to obtain process logics of one sort or another. So result logics which broach the linkages with computing, and what it offers access to, a range of algorithmic, intelligent and other procedures.

4. Towards satisfactory relevant dynamic and process logics

Dynamic logics afford a different direction. Dynamic logics are an outcome of computing theory, a product which, though grounded in features of programming and program verification, borrowed heavily from modal logic, a product accordingly ripe for relevantization. Dynamic logics fit snugly into the process theory sketched. In dynamic logics and the like, programs (or generalising, processes) are the means by which a machine (or wildly generalising, a world) changes from one state to another. In the semantics, processes are represented by two-place accessibility relations between possible states of some machine (or of some world). The first state is seen as the input into a process, the other state, to which the state is converted by the process, is the output. Diagrammatically



But setting apart, the diagram is just as for modal logic semantics, and accordingly admits similar relevant refurbishing.

Because there are significant limitations to what dynamic logics can express (for example they are powerless to detail what happens between or outside states), dynamic logics are but a way-station, investigation has proceeded beyond them. Even, so dynamic logics are now a standard prelude to so-called process logics, which amount to tensed dynamic logics (pleonastic as this may appear). Dynamic logics, which can still be taken to capture *aspects* of action, namely computer doings and procedures, will be developed component by component, in much the way that action logics were. Relevant dynamic logics can be regarded either as an independent development from action logic, built on some amenable relevant logic, or as *incorporating* some or all of the action logics already featured.

Relevant dynamic logics, piece by piece

Dynamic logic includes a countable number of necessity and possibility functors, $[\alpha]$ and $\langle\alpha\rangle$ for each α, where α is some type of process or some representation thereof. The main representation usually envisaged is that of a computer program, which can be run an indefinite number of times, but, as far as the logic goes, it could be any sort of procedure, performance, routine, custom, course, strategy, method,.... Although agents or operators running these routines or programs could be introduced, the standard theory avoids such further complexity. Simply, when A is a wff, so is $[\alpha]A$, for each α.

The necessity-style functor $\boxed{\alpha}$ — usually written for convenience in open form $[\alpha]$ — is understood, in combination with statement or state A, as follows:

$[\alpha]A$ iff A holds (or obtains) every time α operates.

In programming interpretations A is sometimes taken as representing the total state of the computer, but here a statemental construal will be assumed (with A among the data retrievable from the output when program α is run in the computer setting, which takes some contextually fixed data file as input). In an effort to supply a more modal construal, $[\alpha]A$ is sometimes given the flawed reading: whenever α [operates] A must hold. A superior modal construal is as a relative necessity, α-relative necessity. In this respect, an appropriate reading is as follows: It is necessitated through α that A, or more fully: It is necessitated through α's operation that A. Correspondingly, the possibility functor $\langle\beta\rangle$— opened out to $\langle\beta\rangle$ — is construed in combination with A, thus: $\langle\beta\rangle A$ iff it is possibilified through β's operation that A, or, removing the jargon: A holds sometimes β operates.[17] In what follows, emphasis will be upon process-relative necessity, as in $[\alpha]A$; relative possibility can often be defined in a familiar way: $\langle\beta\rangle A =_{Df} \sim [\beta] \sim A$. $\langle\beta\rangle$ will be carried through this definition in what follows.

The basic relevant logic of α-relative necessity, $[\alpha]$, is just that for systemic functors of necessity type: namely

RC. $A \to B / [\alpha]A \to [\alpha]B$
C. $[\alpha]A \,\&\, [\alpha]B \to [\alpha](A \,\&\, B)$

Then initial *programming* postulates flow from the character of programs. In the established modal theory (e.g. the system PDL of propositional dynamic logic of Harel, p. 512), RC is derived from the pair of (optional) additional principles of $[\alpha]$ distribution over implication and $[\alpha]$ necessitation, namely

X. $[\alpha](A \to B) \to .[\alpha]A \to [\alpha]B$, and
RN. $A / [\alpha]A$

While the first, X, is certainly restrictive, the second, RC, is more than that: it is downright implausible, supposing an unlikely (logical) completeness of programs. Just consider incomplete data files. It certainly fails when applied more generally to processes, such as change of (nonclassical) theories or of belief systems, since these are typically incomplete.

Semantics for this much of relevant dynamic logic is but an application of already developed theory. For each process α, each model structure or frame supplies a two-place relation S_α, on states. In the standard relevant semantics each S_α is subject at least to a hereditariness-ensuring condition, namely where $a \leq b$ and $S_\alpha bc$ then $S_\alpha ac$ (further modelling conditions on S_α correspond to further postulates in the familiar way). Then $[\alpha]A$ is evaluated it as follows:

$I([\alpha]A, b) = 1$, i.e. $[\alpha]A$ holds at b, iff for every c such that

[17]This in turn is open to various interpretations, but no doubt it should not be restricted to actual operations of β (e.g. a program may never in fact be run). Rather all, mostly unactualized, operations of β are envisaged, so potential runnings are included.
Notice that the potentialities of the symbolism are by no means exhausted: it is an easy step to $[\alpha|$ and $|\alpha]$ ("right" necessity), and so on.

$S_\alpha bc, I(A,c) = 1$, i.e. iff for every c S_α-accessible from b A holds at c. Modelling conditions for the optional extras of modal dynamic logic are as follows:
for X: where $Rabc$ & $S_\alpha cd$ then, for some x and y, $S_\alpha ax$ & $S_\alpha by$ & $Rxyd$ (i.e. w4);
for RN: where $S_\alpha ax$ then, for some y in y in O, $y \leq a$, for x in O, i.e. x regular (i.e. rw)
(for details see PLI p. 276; the bracketed labels are from there).

Distinctive dynamical operations on program processes

So far relevant dynamic logic has involved nothing but addition of many necessity-type functors (it is thus nothing but the multiple intensional logics of PLI in a different guise). Accordingly we now turn to distinctive features of dynamic logics, to two classes of operations: on programs and on coupled statements. In the further elaboration of relevant dynamic logic we have the advantage that some of the ground has already been surveyed (in Fuhrmann p. 180 ff.). Technical details from that survey can be taken as backdrop (to which any sceptical reader doubting some detail of exact formulation can refer), leaving us free to roam more extensively. For the details are not tied essentially to the special situation Fuhrmann professed to be concerned with, namely 'the modal logic of theory change'; the change or process need not be that of *theory*, nor is the logic *modal* (nonetheless the details are not vastly different from those for the established modal theory, as presented in Harel, upon which what follows is again variational drill).

An elementary program is a sequence of rules, or commands, converting one set of data (input) to another (output), perhaps identical to it as with the degenerate *identity* program, 1, which "copies" input to output. For a theory change story, which will be carried as a special case, each nondegenerate program is an *update* program, transforming one theory into another, changed, one; each proceeds by expansion (adding statements, perhaps one at a time), contraction (subtracting statements) and revision.

Of the operations on or yielding processes, drawn from experience with compound programs, all but one of those usually admitted are easily accommodated within relevant dynamic logics. (The odd operation out in this is indefinite repetition.) The operations are these:

; sequencing. Where α and β and processes, so is $\alpha;\beta$; namely, that consisting of α followed by β. In command form, $\alpha;\beta$ is read: perform α followed by β, or: first run α then run β! While the formation rule always appears to make reasonable sense for programs, for processes it is not quite so clear, for instance where the processes are entirely disparate.

$\cup$ **alternation**. Where α and β are processes, so is $\alpha \cup \beta$; namely that consisting of α or β. In command form, $\alpha \cup \beta$ is read: perform α or β. Thus $[\alpha \cup \beta]A$ is going to assert that A holds every time either α or β operates.

Since α and β are represented semantically, in each frame, by relations, these compounds will naturally be represented by operations on relations. Namely, each will be represented by the obviously corresponding one from relation algebra; ; by relation product and $\cup$ by relation union. That is, $S_{\alpha;\beta}$ is S_α/S_β and $S_{\alpha\cup\beta}$ is $S_\alpha \cup S_\beta$ (on the logical properties of both of which see PM). Outside the setting of decidable programming, such correspondences at once suggest expanding process operations to match significant parts of, or all of, a relation algebra (e.g. positive relation algebra; De Morgan relation algebra, if negative processes can be made good sense of). Since there are pressures to enlarge dynamic logic, to take account of further operations, we shall revert to this issue.

? query. Where A is a wff (statement, or state), then $?A$ is a process. Thus ? is not like ; and $\cup$, an operation on processes yielding a process. It is a function on statements. The intended interpretation of $?A$ is: proceed when A holds ; otherwise fail. But this is pushed down to a material construal, as the usual axiomatic and semantical conditions for $[?A]B$ reveal : it amounts to $A \supset B$. The adequacy of this construal to a real-life notion of querying is quite another matter.

Adding these functions to relevant dynamic logic as so far elaborated is not a demanding matter; it is rather like adjoining functions to a predicate calculus. In fact it is *almost* definitional (but, for one thing, the contexts where the functions are defined exceed those where definitional constraints operate). $[\alpha;\beta]A$ amounts to $[\alpha][\beta]A$; $[\alpha\cup\beta]A$ to $[\alpha]A$ & $[\beta]A$, and in uniform form, $\langle\alpha \cup \beta\rangle A$ to $\langle\alpha\rangle A \vee \langle\beta\rangle A$; and $[A?]B$ to $\sim A \vee B$, and so $\langle A?\rangle B$ to A & B. Now to get down to some solid axiomatic and semantic pay-dirt. The functions are governed by the following controlling axiom schemes, one apiece:

$$\begin{array}{rcl} [\alpha;\beta]A & \leftrightarrow & [\alpha][\beta]A \\ [\alpha\cup\beta]A & \leftrightarrow & [\alpha]A \,\&\, [\beta]A \\ [A?]B & \leftrightarrow & \sim A \vee B \end{array}$$

The semantical modelling conditions for the first two schemes are given precisely by the relational algebraic conditions:
$S_{\alpha;\beta}ab$ iff $S_\alpha/S_\beta ab$, $S_{\alpha\cup\beta}ab$ iff $S_\alpha ab$ or $S_\beta ab$, for all situations or states a, b in K.
To validate a coimplication $C \leftrightarrow D$ it is enough to show that $I(C,c) = I(D,c)$ for every c in K. Now, to illustrate with the first scheme:

$I([\alpha;\beta]A, a) = 1$	iff, for every b such that $S_{\alpha;\beta}ab, I(A, b) = 1$
	iff, for every b such that $S_\alpha/S_\beta ab, I(A, b) = 1$
	iff, for every b and x such that $S_\alpha ax$, and $S_\beta xb$, $I(A, b) = 1$
	iff, for every x such that $S_\alpha ax$, and for every b such that $S_\beta xb$, $I(A, b) = 1$
	iff, for every x such that $S_\alpha ax, I([\beta]A, x) = 1$
	iff $I([\alpha][\beta]A, a) = 1$.

Thus is soundness established. For completeness, simply define canonical $S_{\alpha;\beta}$ through S_α/S_β. Similarly for alternation (with the argument for soundness resoundingly classical). Adequacy is not so straightforward for the next stages of elaboration of relevant dynamic logics.

In established propositional dynamic logic there is one further operation, a perform-repeatedly or a perform-an-arbitrary-finite-number-of-times function, usually written *, but here symbolised #. This operation can evidently be explained through sequencing, because any given program α, it amounts to running α n times, to $(\ldots(\alpha;\alpha)\ldots;\alpha);\alpha$ with α occurring n times, for some $n \geq 0$. Specifically α^n is defined inductively as follows:

$\alpha^0 = 1$, i.e. the identity program (introduced below), which leaves things as they were.

$\alpha^{n+1} = \alpha^n;\alpha$.

Thus $\alpha^1 = \alpha^0;\ \alpha = 1;\alpha = \alpha$, by virtue of the properties of 1. # is deliberately but arbitrarily defined to allow among an arbitrary number of times, no times. Then $\alpha^\#$ is α^n for some finite n. The properties of $\alpha^\#$ in a relevant setting have yet to be worked out satisfactorily. So do other additions prominent on the margins of the standard theory, for instance those which may exceed linear complexity such as parallel processing. Let us proceed to interesting angles that have been worked out relevantly.

Relevant theory change within a dynamic framework

Theory concerns the transformation of one theory into another, changed theory, a froming of an "input" theory to an "output" theory. Similarly for change of propositional systems more generally. Such transformations amount of course to processes, processes of a sort that appear representable in such program-atic formalisations of reasoning about processes as dynamic logics. 'Thus, a natural idea is to explore the prospects for a special kind of dynamic logic: a [multiply intensional] logic with a set of theory change operators' (Fuhrmann p. 181, to whom this idea naturally occurred). In these interesting terms, a logic of theory change becomes an extension of dynamic logic.

The now established (classical) logic of theory change focusses upon operations of expansion, contraction and revision of theories. As revision gets defined (though not uncontroversiall) in terms of expansion and contraction we can concentrate on these. In the established way we shall regard expansion and contraction as operating step by step, addition or subtraction of one wff at a time. The expression $+A$, applying expansion operation $+$ to wff A, indicates the addition of A, while expression $-B$ applying contraction operation $-$ to wff B the subtraction of B. Addition *to* what? Subtraction *from* what? In the established setting these would by be coupled with some theory T, with $T + A$ giving the theory expanding T by A, and $T - B$ the theory resulting by subtraction of B from T. But here, in the amending setting of computer programming they are presumably concatenated with programs or process expressions $\alpha, \beta, \ldots$, giving $\alpha + A, \beta - B$, etc.

A critical question is: What do such expressions mean? And, hardly independently, to what postulates do these confirm? In the confined setting of dynamic logic, program expressions only occur in generalised modal contexts of the form [ ]. So a first part to the question is: how do the likes of $[\alpha + A]$ and $[\beta - B]$ behave? There are two approaches: Fuhrmann's and a more freewheeling approach. On Fuhrmann's approach $+A$ and $-B$ are programs in their own right, and the invariable combinations, of the form $\alpha + A$ and $\beta - B$ are obtained by sequencing, i.e. $\alpha + A$ should be $\alpha; +A$ and $\beta - B$ $\beta; -B$. Thus expansion and contraction operations, $+$ and $-$, are, in formational respects, like query ? Applied to wff, they yield processes: $+A$ and $-B$ are process expressions, signifying primitive processes. Now the intended program interpretation of $+A$ is: proceed to add A to a program; that of $-B$ is: proceed to subtract B from a program.[18] According to Fuhrmann's approach, we obtain this effect by a sequence operation: in $\alpha + A$ i.e. $\alpha; +A$, by running $+A$ after α, and in $\beta - B$ i.e. $\beta; -B$, by running $-B$ after β. But this seems wrong. For $-B$ might have an effect *on* β, not just sequence in after β. Indeed in the established theory setting subtracting B has considerable effects on the theory from which it is removed.

A more *flexible* alternative, which avoids this seeming absurdity, construes $\alpha + A$ and $\beta - B$ not in terms of sequencing but as further programs obtained from programs α and β and wff A and B. Where α and β are appropriate process expressions and A and B are wff then $\alpha + A$ and $\beta - B$

[18]These operations, while applying more extensively than programs, to *theory-processing*, of course do not make good sense when extended to processes not involving statemental data.

are further process expressions. As it happens, this flexible approach does not exclude Fuhrmann's. For A may be added at the end of a data file, and $\alpha + A$ then taken as a convenient abbreviation for $\alpha; +A$. Moreover the approach allows for ready generalisation. For, with but few constraints, what is added or subtracted can be almost any old junk: sets of wff, further programs, etc.

+ expansion. For + to parallel standard conditions for finite expansion of theories, it should conform to at least the following schemes:

E1^0. $[\alpha]B \to [\beta]B/[\alpha + A]B \to [\beta + A]B$.
E2. $[\alpha + A]A$
E3. $A \leftrightarrow B/[\alpha + A]C \to [\alpha + B]C$

Here E1^0 is intended to reflect monotonicity of expression, in that when $T_1 \subseteq T_2$ then $T_1 + A \subseteq T_2 + A$. In fact E1^0 is a rule version (an S2 version) of the usual implicational (or S3) formulation; but it is a formulation with genuine implication in place of material-implication. E2 expresses a success requirement for expansions, while E3 ensures that expansion depends only on the logical strength of wff added. Matching modelling conditions for these postulates, within semantics for relevant dynamic logic, are as follows:

e1^0. when $S_\beta \subseteq S_\alpha$ then $S_{\beta+A} \subseteq S_{\alpha+A}$.
e2. $S_{\alpha+A}x \subseteq |A|$, for $x \in$ O,

where $S_\gamma d = \{c : S_\gamma dc\}$ and $|A| = \{d : I(A, d) = 1\}$ i.e. the range of A.

e3. where $|A| = |B|$ then $S_{\alpha+A} = S_{\alpha+B}$.

These postulates and their matching modelling conditions, still in an early fluid state, admit of much variation. Rather obviously, E3 can, and perhaps should be, strengthened to

E3$^+$. $A \to B/[\alpha + A]C \to [\alpha + B]C$, and correspondingly
e3$^+$. where $|A| \subseteq |B|$ then $S_{\alpha+A} \subseteq S_{\alpha+B}$.

But so far the analysis, particularly the semantics which substantially copies the postulates, is not sufficiently penetrating to help in reducing such fluidity.

− contraction. For − to parallel standard conditions for finite contraction of theories, it should conform to at least these schemes:

C1. $[\alpha - A]B \rightarrow [\alpha]B$
C2. $\sim [\alpha - A]A$
C3. $A \leftrightarrow B / [\alpha - A]C \rightarrow [\beta - B]C$
C4^0 $\sim [\alpha]A\,[\alpha]B \rightarrow [\alpha - A]B$

These amount respectively to rendition, within a dynamic logic setting, of the postulates of inclusion, success, preservation of logical strength, and vacuity. The remaining standard postulate for contraction according to which a contracted set of data should still be a theory, is effectively quaranteed by the programmatic postulates of dynamic logic. Matching modelling conditions for the postulates look like this:

c1. $S_\alpha \subseteq S_{\alpha - A}$
c2. $S_{\alpha - A}x^* \not\subseteq |A|$ for $x \in O$
c3. where $|A| = |B|$ then $S_{\alpha - A} = S_{\alpha - B}$
c4^0. where $S_\alpha x^* \not\subseteq |A|$ for $x \in O$, then $S_{\alpha - A} \subseteq S_\alpha$

As with expansion so here much variation is possible, for instance, strengthening of the premiss of C3 to a one-way implication.

The identity process 1 is a degenerate process which leaves everything as it was. In program terms it

> simply copies input into output — nothing changes. Were we only interested in what *holds* according to a given set of data, we could do without the identity program and just let A express that A hold (i.e. is an item in the file). But we are also interested in what does *not hold*, and we have not made the classical imposition that sets of data are always complete and consistent with respect to the language in which they are cast. Hence, to say — with respect to a set of data — that $\sim A$ holds, is not equivalent to saying that A is absent from the data set in question. We can however express the latter fact by means of the formula $\sim [1]A$. Thus, the addition of the identity program 1 makes an essential contribution to the expressive power of our system (Fuhrmann p. 183, early version, symbols adjusted).

In the logic, 1 is a primitive process constant. It is governed by the following two axiom schemes:

I1. $[\alpha; 1]A \leftrightarrow [\alpha]A$
I2. $[1; \alpha]A \leftrightarrow [\alpha]A$

Since 1 is a process, it is represented "dynamically" by a corresponding relation S_1, which is subject to two modelling conditions matching the axiom schemes:

i1. $S_{\alpha;1} = S_\alpha$
i2. $S_{1;\alpha} = S_\alpha$

Evidently where adequacy, i.e. soundness and completeness, is already established, it can be extended to encompass 1.

It would be a mistake to be too charmed by a comparison of theories with programs, and accordingly of theory change with an elaboration of dynamic logic. For the *disanalogies* are important. For instance, for a fixed computing task, programs can be correct; alternatively they may be unsatisfactory because they omit cases, or because they do not terminate, but loop or similar. Theories do not exhibit analogous features. Conditions for equivalence of programs and of theories differ considerably. While theories can change, they do not, like program, run. And so on. Theory change is it own thing, and not really a kind or extension of dynamic logic.

Nor does dynamic logic succeed in capturing processes. A lesser reason for this derives from the limited expressive power of dynamic logic for programming applications, limitations which so-called process logic (hereafter distinguished as *proceis* logic) aims to overcome through tensing and path analysis. A more important reason, which such proceis logics do not satisfactorily surmount, is the limited way in which program expressions, which stand in for process expressions, enter into the formalism. For in dynamic logic they are essentially confined to appearances within modal bracketings. An additional indicative reason, this time noticed in proceis logics, is the limited adequacy of the modellings of processes in dynamic logics, simply through (static) relations on states (of affairs); at least newer proceis logics apply path analysis, typically considering relations on "paths", i.e. sequences of states.

Toward relevant proceis and temporal logics

Modal proceis logic were developed to surmount certain limitations of dynamic logics, notably inability to accommodate "progressive behaviour of programs", such as what happens before, during and after them. A typical example is provided by the expression, "during the computation, variable z assumed the value 1". Programming functors absent from dynamic logics, because of their insufficient expansive power, include: *during* α, A; *first up* A; *throughout* α, A; α *preserves* A; *it will happen that* A; A *until* B.

Syntactically proceis logics simply add to dynamic (or action) logic some selection of such functors. For example, Harel and coworkers suppose they have supplied a complete system of these functors with their functors ***f*** (for *first*) and ***suf*** (an analogue of *until*) because, by virtue of the completeness of associated theories of linear order, 'all purely temporal connectives are expressible in terms of the U operator', *until* (Harel and Kozen p. 146; Harel p. 595). There are reasonable grounds for doubt as to the adequacy of such claims; after all non-linear theories of time have attracted much investigation in tense logic. Corresponding, impressions of functional completeness of modal proceis logic should be treated with some scepticism.

It is evident that at some at least of the functors cited can be accommodated through the resources of tense logic, or of tense logic combined with dynamic logic — what now gets called "temporal logic". Upon combining relevant tense logics (already treated in PLI) with relevant dynamic logics as above, relevant temporal logics are on offer, requiring little or no new work. So what is new? What is said to really distinguish these proceis logics is semantical in character (thus e.g. Harel p. 597, Segerberg). Semantically proceis include some path analysis. The basic idea is that interpretation $I(A, c)$ of A at c evaluates A not in, or just in, states or worlds, but at paths of states (where paths are defined as before as finite, or countable, sequences of states). Thus elements like c are assigned a specific structure, and relations upon these elements can absorb and reflect some of that structure, undoubtedly a way to try to gain some genuine dynamic character. (Parenthetical remarks: The idea of such structure on worlds is hardly news. For example, under operational semantics for relevant and modal logics, the elements concatenate algebraically in a way conforming to enriched lattice structures: see RLR. Furthermore with higher degree relevant semantics a certain concatenation, fusion, of elements of world set K already occurs. It is more or less evident then that in proceis logics K can be considered as usual as a set of worlds with however a certain sequential structure, with worlds like world-lines. In any case it is known from universal semantics that operational structure of elements can be traded in for relational structure on elements. So the advantages of elements with appropriate structure appear at most pragmatic: perhaps simpler, less devious, and more tractable semantical theory. But here even these advantages are dubious, as will soon appear.)

A interpretation strategy, supplanting states by paths of states can of course be immediately transferred to relevant logic semantics. There are, moreover, major reasons for relevantization:

- the sorts of reasons already offered (e.g. in PLI and RLR) for relevant tense and other logics operate: namely that modal formulations collapse significant distinctions and induce extensive and unnecessary paradox.

- reasons of inconsistency, and also incompleteness. A computer may circuit through inconsistent intermediate states, for example where its data becomes inconsistent. A modal theory cannot satisfactorily treat inconsistent information, should it arise, as it well may, during processing; a relevant theory can.

As with most other extended modal functors, so with proceis functors, relevant revamping can simply assume straightforward modal evaluation rules. Of course, though the schemes look alike, they do not mean the same, because of the wider class of states and paths comprehended.

U for *until*: U is syntactically a two-place statemental functor, with $A \,\mathrm{U}\, B$ read: A *until* B. An appropriate evaluation rule for U within the path setting appears to proceed as follows: $I(A\,U\,B, p) = 1$, i.e. A *until* B holds at path p, iff there is a state y along path p satisfying B such that all states occurring on path p before y satisfy A, i.e., introducing some symbols, for some y in $p, I(B, y) = 1$ and for x in p such that $x < y$ $I(A, x) = 1$. There are difficulties with this rule (which is that adopted in effect in modal process logic; see Harel and Kozen p. 147), because B may flicker on and off along p. Suppose A is on until B comes on briefly but goes off then and then B goes off for a period; should we say A until B, as the rule obliges? Furthermore, if it *is* legitimate to refine meanings of notions explicated (as it is, but within tighter limits than exact philosophers usually heed), then it also appears permissible to abandon paths and revert to the ways of temporal logic.

Define *routes* just through a familiar (route-)accessibility relation, P say, on worlds. State a is on a route from b iff b is P-accessible from A, i.e. Pba. A route from a is whatever is P-accessible from a. For proper routes P will be expected to satisfy requisite conditions. Then in *pure world* terms,

$I(A\,U\,B, a) = 1$ iff for some y such that $Pya\, I(B, y) = 1$ and for every x such that $x < y\, I(A, x) = 1$.

Thereupon results, so to say, an utterly commonplace mixed-accessibility-relations modality: A *until* B holds iff B is route "possible" and relative thereto A is temporally "necessary". The relevant treatment thereupon fits neatly onto already developed theory (that of PLI p. 276 ff.).

Similar transformations can be effected for other functors of proceis logic. An example or so:

F for *first* (*up*): Syntactically F is a one-place statemental functor, which does not make good sense outside a suitable path or route setting. In path terms, $I(\mathrm{F}A,p) = 1$ iff A holds at the initial state of path p. Generalising to states and routes from them,

$I(\mathrm{F}A,a) = 1$ iff for some b, b is initial along the route P-accessible from a and $I(A,b) = 1$, where b is *initial* if b precedes all other states along that route (and is not preceded by any.).

A different class of functors, explicitly involving processes and directly complementing dynamic logics, comprises the series *before, during, throughout* and *after*.

b for *before*: Syntactically **b** is a two-place functor on process expression and statemental forms, with $\mathbf{b}\,\alpha\,A$ reading: *before* $\alpha\,A$. What this means in an actual world looks straightforward: at some state c, preceding the initial state of α, A holds at c. Troubles begin beyond the actual, in which semantics is inevitably embroiled. One way to reach an evaluation rule for $I(\mathbf{b}\alpha\,A,a) = 1$, i.e. *before* $\alpha\,A$ holds at a, world-relativizes the happening functor $\boldsymbol{H}$ (a necessary stage of advancement in any case), or an amenable variation on that, such as : at a α projects route r, condensed $\mathrm{r}(\alpha,a)$. Initial states of routes are as before. Then

$I(\mathbf{b}\alpha\,A,a) = 1$ iff $\mathbf{r}(\alpha,a)$ and for some c preceding an initial state of $\mathbf{r}, I(A,c) = 1$.

a for *after*: The detail is a reverse image of the preceding. Similarly substitute: succeeding a terminal state of **r**. Again adequate relevant axiomatisations remain to be investigated.

d for *during*: It can be assessed like a kind of route possibility.

$I(\mathbf{d}\alpha\,A,c) = 1$ iff $\mathbf{r}(\alpha,a)$ and for some c in $\mathbf{r}, I(A,c) = 1$.

t for *throughout*: This resembles a kind of route necessity.

$I(\mathbf{t}\alpha\,A,a) = 1$ iff $\mathbf{r}(\alpha,a)$ and for every c in $\mathbf{r}, I(A,c) = 1$.

There is an uneasy, but hopefully unproblematic, nesting of worlds entangled with these proposals. Namely, a process has for each world a routing which itself consists of strings of worlds: worlds within worlds, so to say.

Though the flourish of activity on modal proceis logics (peaking in the early 80s) has subsided, proceis logics remain very much in developmental stages, relevant varieties in particular. Already visible, however, are sundry problems with such logics. First, there appear to be too many functors, which it is difficult to discriminate between satisfactorily (witness the families of functors, one gross family for "during" for instance, shunted out in Segerberg 84 p. 18). Further, not utterly disconnected, there is nowhere in sight, so far, a decent set of canonical functors around which to organise the theory, by contrast even with dynamic logic. But a much more damaging criticism is that such logics only obliquely include their alleged objects, processes. It is not just, or at all, that path analysis, the main approach is (as explained earlier) deficient; it is that the path procedure only affords a backdoor, semantical, way of adverting to processes. Syntactically process expressions are always covered or boxed around by modal operators; they are never free, never subjects of discourse. While it is no doubt imagined that this represents ontologically prudent practice, what it really represents is logical practice emasculating a genuine process theory. In a proper theory processes are freely *there*, to put it picturesquely, in the logic: processes nakedly process, and cavort.

A next stage of advancement towards genuine process logic takes processes as objects (and therefore as objects of neutral quantification), and correspondingly their expressions as proper subjects of discourse, and accordingly included as such in the syntactical theory. Thus much of what in process theory has been shunted into the semantics should be translocated within the logics (for instance, the nice preliminary theory in Segerberg 90 pp. 11–13). That means joining the De Morgan lattice theory and kategory theory of processes (of part II, where concatenation, upper and lower bounds, etc., are readily defined) with the relevant theory sketched above, and building thereupon and therefrom. Great logical riches beckon.

5. Interim conclusion

The conclusion has to be that we have not reached a conclusion. We have simply reached a stopping point, dictated by available time and energy. More is required than variational drill on modal elaboration. Locating satisfactory process logics lies in the future.

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