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ESSAY REVIEW

Making space for fundamentals

Frank Arntzenius (with a contribution by Cian Dorr): Space, time and stuff. Oxford and New York: Oxford University Press, 2012, viii+288pp, £30.00/US $55.00 HB

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Frank Arntzenius says this book is not primarily concerned with overarching themes and that his main goal is to examine some loosely related topics. These all have to do with what theories of modern physics imply (or at least suggest) about the structure and contents of space and time. Each of the eight chapters focuses on a different potential implication and may be read independently. But the author does acknowledge that two overarching themes emerge.

The first theme is that much of modern physics is best understood as positing novel spaces—physical objects distinct from ordinary 3-dimensional space (or the 4-dimensional space–time of relativity) while sharing some elements of geometric structure. The second is that "our knowledge of the structure of the world derives from one basic idea: the idea that the laws of the world are simple in terms of the fundamental objects and predicates". I will return to these themes after discussing the contents of each chapter.

After a gentle introduction to the structure postulated for time by Newtonian and then relativistic physics, Arntzenius devotes the rest of Chapter 1 to explaining and then assessing the prospects of the view that time has no structure. Basically Julian Barbour's view, the idea is that its metric, topological and order structure are not fundamental features of time, but rather derive from non-temporal relations among what happens at each moment. Using minimal mathematics, Arntzenius explains how to formulate Barbourian versions of Newtonian, special relativistic and general relativistic space–time theories. After showing why none of these is clearly empirically inferior to a standard version formulated within a relatively rich temporal structure, he presents and assesses objections to each. Barbourian general relativity faces the strongest such objections. Barbour himself expresses confidence that further development of his programme to encompass a combination of general relativity with quantum theory will overcome them. But Arntzenius declines to pursue his

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examination in such a speculative and mathematically intractable direction. He concludes simply that the idea that time has no structure is surprisingly viable.

Chapter 2 is a more substantial and original contribution to natural philosophy. In his doctrine of Humean supervenience, David Lewis famously touted the view that, together with a set of point-like entities bearing spatiotemporal relations to one another, all there fundamentally is to the world is an assignment of perfectly natural intrinsic properties at space–time points. While Lewis thought this doctrine meshes well with classical physics, a broad consensus has emerged that it conflicts with failures of separability associated with quantum entanglement. But Arntzenius argues that classical physics already conflicts with the "point-wise" separability of Humean supervenience. Instead, he proposes a slightly relaxed condition of *neighbourhood separability* and goes on to show how not only space–time structure but also world histories compatible with classical theories of particle motion and propagating fields may be neighbourhood-separable.

What is the difference between separability and neighbourhood separability, and why does it matter? Arntzenius follows Lewis in cashing out separability in terms of supervenience or determination. He says that a world history is separable just in case all of its features are determined by intrinsic features of each space–time point. But that cannot be his official definition since he sides with Lewis in including in the supervenience base geometric features of space–time that are not intrinsic to each space–time point. One such feature is space–time topology: space–times may have different topologies even though every space–time point in each shares the same intrinsic topological properties (viz. that its singleton set is closed rather than open). But topology specifies neighbourhood relations among space–time points and so permits a definition of neighbourhood separability. Arntzenius introduces this notion in its first application to topology itself: "topology is separable in the sense that given any division of the total space into open neighbourhoods, the topology of each of the neighbourhoods determines the total topology" (47). (His "trivial" proof of this does not make the nature of the determination relation fully explicit.) He points out difficulties in applying Lewis's idea of taking spatiotemporal distance relations between points as primitive in general relativity, and then shows how even here the metric structure underlying such distance relations is neighbourhood-separable.

If world history is not separable, then even if the world is composed of point-like parts what it is like depends on more than their intrinsic properties and external relations. But if it is neighbourhood-separable, then world history is nevertheless determined by what happens as close as you like to each such part. Arntzenius goes on to show that whether even that is true depends not just on what physics says about the contents of space–time but also on how that is parsed. In a fascinating discussion, he presents and evaluates alternative analyses of notions of mass, velocity, fields and locality as these figure in classical physics before tentatively concluding that classical worlds are (only) neighbourhood-separable.

Chapter 3 is anomalous in several respects. Unlike the rest of the book, it does not focus on classical physics: indeed, it assumes knowledge of quantum mechanics comparatively more detailed than the acquaintance with classical physics assumed in most of the other chapters. Despite its title, it concerns only the world's *fundamental* structure, with no regard to the pressing question as to how this could possibly support the rest. Most important, what Arntzenius says about this fundamental structure is hard to square with the book's general line on fundamentality, which receives its fullest expression in Chapter 8.

Arntzenius first presents a standard argument that a quantum world is neither separable nor neighbourhood-separable. He then runs through a menu of currently fashionable proposals by David Albert and others involving the idea that the fundamental structure of a quantum world is most perspicuously presented in terms of some high-dimensional configuration space rather than physical space(–time). Finding these neither palatable nor capable of restoring (even) neighbourhood separability, he judges Wallace and Timpson's space–time (density operator) state realism an acceptable resting point despite its non-separability. But he presses on to tout the virtues of a view he calls Heisenberg-operator realism extracted from work of Deutsch and Hayden, including its allegedly miraculous ability to portray the quantum world as fundamentally separable and also dynamically local.

The Heisenberg picture represents the evolution of a system by taking the operator representing an observable to be explicitly time dependent, unlike the quantum state on which this acts. Physicists standardly regard choice of the Heisenberg picture over the Schrödinger picture as purely a matter of convenience since they yield the same expectation values for all observables. But for the Heisenberg-operator realist, only the Heisenberg picture faithfully portrays the fundamental nature of the quantum world. Since Heisenberg operators in a relativistic quantum field theory may be assigned to arbitrary open sets of space–time points (if not point-wise), this fundamental reality is (at least) neighbourhood-separable. Moreover, since these operators obey local differential equations, the Heisenberg picture portrays a dynamically local world.

But only those distracted by mathematical sleight of hand will believe the Heisenberg-operator realist has succeeded in pulling the rabbits of separability and locality out of the quantum hat. His proposal looks like a non-starter because even in the Heisenberg picture the actual (unchanging) quantum state lays at least equal claim with the operators to represent quantum reality. But suppose one begins by making a purely conventional choice of quantum state vector. This will be related to the actual quantum state by a unitary transformation U. Consider some Heisenberg operator A in the Heisenberg picture based on the actual quantum state. The operator $A' = UAU^{\dagger}$ has the same expectation value in the conventionally chosen state as does U in the actual state. So a Heisenberg-operator realist can regard the choice of fixed state as purely conventional, putting all the burden of representing the actual local condition of a quantum world on the Heisenberg operator A'.

But Arntzenius has given us no reason to believe a neighbourhood-separable assignment of Heisenberg operators like A' is capable of shouldering this burden. First, A' generically encodes information about joint expectation values involving observables assigned to open sets nowhere near where A' is assigned. It is able to do this in so far as the transformation U implicitly encoded the non-separability of the *actual* quantum state even if the arbitrarily chosen fixed state was separable. So if the proposed assignment of Heisenberg operators like A' *does* represent features of a quantum world, these features do not concern just what the world is like in the neighbourhood where each such operator is assigned, and nor are they neighbourhood-separable.

Elsewhere in the book, Arntzenius himself provides his reader with all the materials for a deeper objection to Heisenberg-operator realism. In section 2.6, he correctly diagnosed one source of the mistaken belief that a classical world is separable in the fact that classical mathematical objects such as mass densities, metric and curvature tensor are defined point-wise. And Chapter 8 is a systematic attempt to unpack the unobvious non-mathematical representational content of mathematical objects as applied in physics. Yet he concludes Chapter 3 by supporting a view (Heisenberg-operator realism) that presumes that certain highly abstract mathematical objects represent separable fundamental features of reality because they are assigned at (or near) space–time points.

Chapters 4 and 5 address the ontology of space–time. Some metaphysicians seem fascinated by the idea that space and/or time may lack least parts—points. Arntzenius explores the prospects of pointless spaces and concludes that it is not clear whether we can develop simple physical theories that would lend credence to this idea. He offers space–time substantivalists several ways to respond to Leibnizian objections to their view while arguing that substantivalism gives rise to simpler and more natural theories than relationism. I learnt something by reading these chapters, but did not find them ground-breaking.

By contrast, Chapter 7 essays a radical reinterpretation of the celebrated CPT theorem of quantum field theory according to which the theorem has little to do with charge but indirectly tells us something important about the structure of space–time. In this chapter, Arntzenius credits work of Hilary Greaves: in his preface, he says she should have written it! Each of C, P, T stands for an operation on a system that (respectively): changes the sign of all charges in a system (more generally, interchanges particles and antiparticles); switches the spatial “handedness” of everything in the system; reverses the time order of all processes in the system. The CPT theorem is usually understood to show that any system of quantum fields is invariant under the successive application of these three operations (in any order). Arntzenius here explores the idea that this is a misunderstanding of what the theorem says: that the operation conventionally denoted as “CT” should really be understood as the time-reversal operation instead of the operation conventionally denoted by “T”, so, correctly interpreted, the theorem says that a quantum field system remains invariant under a change in spatiotemporal orientation (whether or not it remains severally invariant under spatial and temporal reorientations). If that is what the theorem says, then it is telling us that fundamental physics gives us no reason to believe that the structure of space–time includes a preferred orientation. Arntzenius could then recommend that we not believe this.

In fact, he makes no such direct recommendation for a reason that takes us back to what he said about quantum mechanics in Chapter 3. If one takes the quantum state as fundamental (as do Wallace and Timpson), then that is indeed what he would recommend. But if it is rather quantum field operators that represent fundamental reality (as a Heisenberg-operator realist maintains), then a *further* operation equivalent to C is required to ensure invariance of a system of quantum fields, so we get back to the conventional interpretation of the theorem as indeed a

CPT theorem, which then has no implications for space–time orientability. Once again Chapter 3′s view of the quantum world is hard to reconcile with the picture painted in the rest of the book.

Chapter 6 argues that some fibre bundle spaces associated with gauge theories are just as real as the physical space(–time) they incorporate. Arntzenius uses this to support the standard view that distinct objects can have a property in common while maintaining that no sense can be made of the idea that the same field value obtains at distinct spatiotemporal locations: the objects are (open sets of) bundle points, and the properties correspond to being occupied by the field in question. Maudlin (2007) had taken gauge theories as a counter-example to the standard view of properties, while I (2001) had argued against fibre bundle substantivalism.

Here we encounter both themes Arntzenius highlighted in his introduction. He argues for vector bundle substantivalism and dismisses objections based on intuitions about possibility in the same way he had argued for substantivalism over relationism about space–time in the previous chapter. But he does not consider epistemological and semantic objections analogous to those offered in Chapters 2 and 4 of my (2007). (Admittedly, these were not explicitly directed against vector bundle substantivalism—by then I had come to regard fibre bundle substantivalism as sufficiently implausible not to warrant separate treatment!) He examines what he calls gauge relationism (cf. the holonomy interpretation of my (2007)) and conducts an ultimately fruitless search for a simple formulation of a gauge theory in gauge-invariant terms. For Arntzenius, that is enough to warrant the (provisional) conclusion that gauge theories are best understood as theories about the geometrical and occupation structure of fibre bundle spaces.

But does it? Why should a world lacking fibre bundle spaces conform to simple gauge-invariant laws, and can his preferred formulation of a gauge theory be formulated in laws that "are simple in terms of the fundamental objects and predicates"? Chapter 6 views the fundamental objects of a gauge theory as pure sets in the set-theoretic hierarchy, constructible from the null set alone. Arntzenius is clearly uncomfortable with this Platonism, and in the final chapter, he and Cian Dorr set out on an ambitious nominalist project to purge (at least classical) physics of such abstracta.

Field (1980) presented a nominalistic formulation of Newtonian gravitational theory in support of the view that even science has no need of abstract mathematical objects. The authors of Chapter 8 pursue a two-stage programme to extend this to theories couched in differential geometry. The easier nominalist task is to show how nominalistically to re-express their predicates relating physical to mathematical objects (e.g. "The curvature scalar is zero at space–time point *p*"): the hard nominalist task is to reformulate their laws simply enough to capture the explanatory power of such "mixed" talk (and so to answer a metaphysician's hopes for physics to describe the fundamental intrinsic structure of the world).

In this challenging chapter, Arntzenius and Dorr display considerable technical ingenuity by constructing nominalist surrogates for the mathematics of differential geometry employed so widely in contemporary physics. They tackle the easier task by constructing such surrogates out of peculiar space–time regions, but acknowledge these do not suffice for the harder task. For that they appeal to a richer ontology of spaces, whether a simple scalar value space or a vector bundle space. These go beyond the space(–time) in which theories are generally taken to represent what happens.

But why should we believe in such additional spaces? Scalar value space is brought into service as an allegedly physical substitute for a similarly structured mathematical space. Only distinctively physical evidence could provide a reason to count it as more than an alternative abstract structure. Chapter 6 was supposed to put vector bundles on firmer physical footing, but did not establish how empirical support for any gauge theory could supply the necessary evidence. But suppose on the contrary that the success of a variety of gauge theories *did* warrant belief in a corresponding variety of substantive vector bundle spaces. How would these all be related to one another and to what is naturally thought of as their common base space—space–time? There seems no reason to plunder one rather than another of these supposedly physical spaces to collect building materials for differential geometry and theories that require it instead of sticking with the purely mathematical space that exhibits their shared relevant structure. Still, the authors of this spirited final chapter are to be congratulated for their sustained attack on some deep and difficult philosophical issues of mathematical and physical ontology all too often ignored or treated superficially by physicists.

Despite its challenging subject matter, much of this book flows like an edited transcript of lectures. There are a lot of typographical errors and some mistakes in the figures, but these should not hinder a reader's enjoyment of the freshness of its provocative ideas.

References

Field, Hartry. 1980. *Science without numbers*. Princeton: Princeton University Press.
Healey, Richard. 2001. On the reality of gauge potentials. *Philosophy of Science* 68: 432–455.
Healey, Richard. 2007. *Gauging What's Real*. Oxford: Oxford University Press.
Maudlin, Tim. 2007. *The Metaphysics within Physics*. Oxford: Oxford University Press.