Contingentism and Correctness Conditions for Mathematical Fictionalism

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Introduction

Fictionalism is an anti-realist view in the philosophy of mathematics that takes mathematical statements to be about abstract objects. However, fictionalists reject the existence of abstract objects like numbers. Thus, they consider ordinary mathematical sentences like “2 is even” to be false. Nevertheless, they must provide an account of fictional mathematical correctness in order to explain the importance of mathematics in our reasoning and its applicability in our everyday lives. For this reason, fictionalists have to explain why and under what conditions “2+2=4” is fictionally correct while “2+1=4” is fictionally incorrect, even though they take both of these statements to be strictly speaking false. There are different accounts of fictionalism that provide different explanations of what makes a mathematical sentence fictionally correct. However, we might wonder to what extent those correctness conditions can be objective if mathematics is merely a useful fiction—is mathematical correctness something independent of people?

Imagine a possible world in which no people exist. In this world, is the sentence “3 is prime” correct? Zoltan Szabó has argued that fictionalists about mathematics will have to answer the question in the negative, because fictionalism renders fictional mathematical correctness conditions contingent on the existence of sentence tokens and therefore people. However, common intuition suggests that “3 is prime” would be correct even in a world with no people. This conclusion can be secured by adopting a form of contingent fictionalism that is different from the one Szabó has considered. In this paper, I will argue that there is a version of fictionalism in the literature, due to Mark Balaguer, that is capable of avoiding Szabó's objection by employing a different kind of contingentism that does not make fictional correctness dependent on the existence of linguistic communities; in virtue of this, this version of fictionalism maintains the objectivity of mathematics.

In Section I, I will briefly explain what mathematical fictionalism is. In Section II, I will explain why fictionalists need to rely upon a 'true-in-the-story-of-mathematics' predicate and the role that this predicate plays within fictionalism. I will then articulate two competing accounts of what truth-in-the-story-of-mathematics consists of. In Section III, I will summarize an objection to fictionalism that Szabó poses in the form of a dilemma: either fictionalism cannot escape commitment to abstracta, or it commits to an implausible version of contingentism. In Section IV, I will explain Mark Balaguer's counterfactual reading of 'true-in-the-story-of-mathematics', and explain how fictionalists can avoid Szabó's objection if they use Balaguer's reading of this operator. In Section V, I will compare the correctness conditions provided by each view and argue that the counterfactual account provides a greater degree of objectivity and is capable of avoiding certain theoretical problems inherent in the referential account. In Section VI, I will conclude with a summary of the features that make the counterfactual account plausible.

I. Fictionalism

Fictionalism in mathematics is a metaphysical view about our mathematical discourse that opposes Platonism. Platonists hold that our mathematical statements are about abstract objects, which are non-physical and non-mental. Moreover, these objects do not occur within space-time; call this the *Platonist semantic thesis*. Furthermore, Platonists maintain that abstract objects actually exist; call this the *Platonist ontological thesis*. The primary virtue of Mathematical Platonism is its ability to provide a simple, consistent, and robust semantic analysis of our mathematical discourse. It allows us to explain the objectivity of mathematical correctness, and it allows us to account for the infinity of numbers; if, for instance, numbers were physical things, it would be an empirical question what the largest number was. Moreover, Platonist semantics does not impute implausible semantic intentions onto speakers; it does not suggest, as some anti-Platonist views have, that when a person says "2 is even" that she *means* "if there are numbers, then 2 is even". As such, it can plausibly be understood as providing a *face-value reading* of mathematical discourse. Fictionalists agree with Platonists that if there were numbers, they would be abstract objects. Thus, they are committed to the Platonist semantic thesis; they think that our mathematical sentences are supposed to be *about* abstracta. However, unlike Platonists, fictionalists do not think there are any abstract objects—they reject the Platonist ontological thesis. Fictionalists are a subset of nominalists—they only believe in the existence of concrete (non-abstract, physical or mental, spatiotemporal) objects.

Both fictionalists and Platonists (of the kind I am considering) assume that there is only one kind of existence. The result of this univocal quantificational analysis is that all existent things exist in the same sense of the word. The sentence “3 is prime” is of the logical form **Fa** and consequently entails a statement of the form **(∃x) (Fx),** which quantifies over numbers. Oranges, houses, and numbers are meant to exist in the same sense and there is no substantial difference in the kind of existence involved in those sentences that express existential claims (i.e., “There is a house” and “There are numbers”). Numbers, however, are not concrete objects; they are supposed to be abstract objects and according to fictionalists they do not exist since only concrete objects exist. As such, fictionalists believe that mathematical sentences are false and that the singular terms occurring in mathematical sentences are vacuous and fail to refer[1]. When fictionalists claim that terms fail to refer, they mean that those terms are supposed to be about some object(s), and those objects do not exist. Moreover, fictionalists are committed to the view that the truth-value of the sentences in which these terms occur depends upon the existence of the objects in question. For instance, when somebody says: “That tree has green leaves”, the truth of the statement depends upon the existence of the tree that the person is intending to refer to, there being leaves on that tree, and those leaves being green. Fictionalists take statements like “3 is prime” to function analogously, but they hold that 3 does not exist (because abstract objects do not exist), and so they think the statement is false.

II. TRUTH IN THE STORY

While fictionalists believe that sentences like “3 is prime” are strictly speaking false, they think that there is nonetheless something importantly correct about such sentences; they believe that such sentences are *true-in-the-story-of-mathematics*. This true-in-the-story predicate plays a fundamental role in the fictionalist program; it enables fictionalists to recapture correctness conditions for mathematical statements, to begin explaining the important work that mathematical reasoning does in our everyday lives, and to vindicate our strong intuitions about arithmetic and other branches of mathematics. According to fictionalists, mathematical sentences are false analogous to the way that "Alice drank tea with the Mad Hatter" is false but nonetheless *true in the story of Alice in Wonderland*. The view is that while neither "1+1=2" nor "1+1=3" is true simpliciter, "1+1=2" is true-in-the-story-of-mathematics[2] while "1+1=3" is not (Balaguer 1998, pp. 12-13 and Field 1993, p. 289). Thus, while mathematical sentences are not true simpliciter, there is another truth predicate—true-in-the-story-of-mathematics—which applies to all the mathematical sentences that we typically think are true. Alternatively, if a statement is true-in-the-story-of-mathematics, it is *fictionally correct*. So the true-in-the-story predicate provides *fictional* correctness conditions for mathematical statements. However, we must keep in mind that the *literal* correctness conditions are different from the *fictional* correctness conditions. The former render all mathematical sentences false (or vacuously true), whereas the latter provide another layer of correctness; namely, one that is relativized to the story of mathematics. In what follows, when I speak of correctness conditions, I mean *fictional correctness conditions*.

In providing correctness conditions for mathematical statements, the fictionalist *pragmatically* uses a fictional truth as a nearby truth. The word "*pragmatically*" is used to show that the sentence that carries the fictional truth is not intended to substitute the original sentence as a mere synonym—it is not a *semantic paraphrase* (Szabó 2003, pp. 20-24). The pragmatic paraphrase, which provides us with the result that its original counterpart is true-in-the-story-of-mathematics, diverges in truth-value from the original sentence, which is literally false. Fictionalists in general accept this distinction between semantic and pragmatic paraphrases, and hold that the correctness conditions for fictional truth are provided by pragmatic paraphrases; they disagree with one another about what the pragmatic paraphrases for mathematical sentences *are*. There are several accounts of *truth-in-the-story-of-mathematics*. I will examine two views: the first is a referential account; the second is a counterfactual account. The referential account holds that in talking about the story of mathematics, we refer to a *theory*, whereas the second holds that if the numbers that our mathematical theories purport to be about existed, then certain mathematical sentences would be true.

1. The Referential Account of Fictionalism (RAF): Szabó considers this view when he articulates a possible objection to this interpretation of fictionalistic correctness conditions (Szabó 2003, p. 23). This view takes the true-in-the-story predicate to be equivalent to an *according-to-T* operator. The idea is that, on this view, to say that S is true-in-the-story-T is just to say that according to T, S. On Szabó's construal, T occupies a referential position, so this view entails commitment to a referent, namely, a theory. So fictionalists of this sort hold that "*According to Peano Axioms, every number has a successor"* is true and stands in as the pragmatic paraphrase for the literally false sentence *"Every number has a successor"* to provide fictional correctness conditions.

2. The Counterfactual Account of Fictionalism (CAF): Mark Balaguer's view as described in *Fictionalism, Theft and the Story of Mathematics* (Balaguer 2009) develops a different account of correctness conditions. On this view, we can account for the correctness conditions of mathematics if we give a counterfactual reading to the *true-in-the-story-of-mathematics* predicate. In particular, on this view, to say that some sentence S is true-in-the-story-of-mathematics is to say this: "*If there were abstract objects of the kind that our mathematical theories purport to be about, then S would be true"*. For the purposes of this paper, I will use C: "*If there were abstract objects of the kind that our mathematical theories purport to be about, then 3 would be prime"* as an illustrative instance of the counterfactual that gives correctness conditions for the story of mathematics.[3]

On Balaguer's view, correctness is determined partially by our intentions, as it is a matter of *what our theories purport to be about*: we have certain pre-theoretic beliefs about what we are trying to capture by means of our mathematical theories, and these beliefs put certain constraints upon what can count as consistent with our full conception of numbers, sets, etc. The counterfactual correctness conditions are capable of capturing this fact, and of providing correctness conditions beyond what can be entailed by currently accepted axioms and theorems. The example Balaguer uses to illustrate this point concerns mathematicians who discover a correct answer to the question of whether the continuum hypothesis is true or false, something which is undecidable given currently accepted set theoretic axioms. Neither the continuum hypothesis nor its negation is a consequence of *currently* accepted axioms. However, mathematicians might be able to discover an axiom which they all take to be obvious, and which provides an answer to the continuum hypothesis question that is compatible with currently accepted set theory. If correctness conditions are given by our currently accepted theories, then in this situation, the continuum hypothesis would *become* true-in-the-story-of-mathematics as soon as it is incorporated into the theory. This intuitively seems wrong, as the axiom would express part of the conception of sets that mathematicians had been concerned with all along, and the mathematicians would take this axiom to have been true and the continuum hypothesis to have been correct all along. So, the correctness conditions of mathematics cannot depend solely on currently accepted axioms and theorems but also on our conception of the branch of mathematics we are talking about (e.g., set theory).

III. SZABÓ'S OBJECTION TO FICTIONALISM AND AN OBSCURE ACCOUNT OF CONTINGENTIST FICTIONALISM

According to Szabó, if we interpret *true-in-the-story-of-mathematics* as "*According to T*", this involves reference to a theory and thus fictionalism carries commitments to theories (Szabó 2003, p. 23). That generates an issue as fictionalists must settle on a particular view of what a *theory* is. A theory can either be an abstract object (composed of types) or a concrete object (composed of tokens). On one standard reading, a theory is a set of sentence types, or propositions. A proposition is an abstract object; it bears the meaning of a sentence, and it is what is purported to be held in common between sentences of various languages that are taken to *mean* the same thing. For example, "Snow is white" and "La neige est blanche" mean the same thing, and so express the same proposition. If this is the case though, it cannot help fictionalists, as they would be explaining why some mathematical sentences are correct in virtue of features of some other abstract objects. However, fictionalists reject all abstract objects. If the fictionalist claims that she does not believe that propositions exist either, but that talk of such things is merely a useful fiction (i.e., that it is false), then she would still owe an account of what makes one sentence about propositions correct and another incorrect, and insofar as she tries to provide this account in terms of other abstract objects, her account faces an infinite regress. As such, an account which attempts to provide correctness conditions for talk about abstract objects in terms of other abstract objects will simply conclude that no mathematical sentence is *fictionally* correct, i.e., true-in-the-story-of-mathematics. If the *correctness conditions* refer to abstracta (which, on the fictionalist supposition, do not exist), then *all the sentences* would end up false and there would not be any difference in correctness between the sentences “3 is prime” and “4 is prime”.

Szabó points out that fictionalists can escape this problem by adopting the view that theories are concrete entities, e.g., linguistic tokens like strings of sounds, or piles of ink. But according to Szabó, this is problematic because, given the fact that linguistic tokens do not exist necessarily, the sentences will express merely contingent truths. However, mathematical truth is presumed to be necessary, so we face an issue that needs to be resolved. The particular form of contingentism that is entailed by this interpretation of the *true-in-the-story* predicate is not viable. Nevertheless, as I will argue in the next section, if we follow Balaguer’s interpretation, then we can develop a sensible kind of contingentism, which is capable of maintaining the objectivity of mathematics while providing a plausible account as to why the non-existence of mathematical objects is contingent. The problem with the kind of contingentism that Szabó considers stems from the idea that sentence tokens determine mathematical correctness. If sentence tokens determine mathematical correctness, then the correctness conditions will depend on the existence of people. But it seems that mathematical correctness is not dependent on the existence of people.

To begin illustrating this problem, suppose that there is a world in which there are no people, and consequently no mathematical or linguistic communities, and thus no mathematical practice develops. The statement, "According to T, S" will be false in this world. That is because there are no concrete mathematical theories in this world, since there are no individuals to express theory-tokens. As such, the term T will fail to refer to anything. Given this, there will not be any mathematical statements that would count as true-in-the-story-of-mathematics; both "3 is prime" and "4 is prime" would be false, and neither would be *fictionally* correct. This view thus entails that fictional mathematical correctness is *contingent on the existence* of sentence tokens, and consequently this version of contingentism makes substantive correctness conditions (ones in which some statements are *fictionally* correct) depend on the existence of some mathematically competent linguistic community. This seems intuitively wrong; if it is possible that certain mathematical sentences are correct, then they should be correct at some worlds which do not contain mathematically competent thinkers. According to the contingentist fictionalism (RAF) considered by Szabó, there would be no fictionally correct statements in such worlds. This suggests that the RAF account makes correctness conditions depend on the wrong things.

This version of fictionalism thus commits itself to a dependence thesis that we have no reason to endorse; it says that the fictional correctness of the statement “3 is prime” depends on there being sentence tokens that assert that 3 is prime. A further disadvantage of the RAF account is that it deviates from a face value reading of mathematical sentences by claiming that there are certain inferences that are valid which are intuitively invalid. For instance, the statement “According to PA, 3 is prime” entails that people exist. This inference, however, is wildly implausible. That mathematical statement has not implied anything about people.

Thus, this contingentism that Szabo's objection entails is not viable. However, there is another kind of contingentism, that counterfactual fictionalists could make use of. This kind of contingentism holds that some things do not exist (or exist), however there is nothing which necessitates their non-existence (or existence). To begin illustrating this alternative conception of contingentism, take the following Contingentist Platonist account, developed by Gideon Rosen (2002): mathematical objects exist but not of necessity, and so occur within some worlds, and not within others **(Φ & $\Diamond$-Φ)**. In the worlds in which these objects exist, the correctness conditions for mathematical statements are determined by features of the mathematical objects. Given the plenitude of possibilities, there are worlds in which mathematical objects do not exist and yet the physical world is exactly as it is in *the actual world*. Contingentism, in this sense, presupposes that the concrete and abstract realms exist independently of one another and that it is possible for one to exist without the other. The two classes of things (concreta and abstracta) are utterly distinct from one another and there is no fundamental dependency between the two. The fictionalism developed by Balaguer uses a counterfactual conception of truth-in-the-story-of-mathematics that can be paired with this kind of contingentism that is different from the RAF kind of contingentism.

IV. BALAGUER'S FICTIONALISM AND A VIABLE VERSION OF CONTINGENTIST FICTIONALISM

The correctness conditions, on this view, are given by the counterfactual C: "*If the abstract objects that our theories purport to be about existed, then 3 would be prime".* Balaguer uses a counterfactual conditional and thus avoids the use of declarative sentences that carry ontological commitments to abstracta or concreta in interpreting correctness conditions. The claim is merely that if the world were some certain way—in particular, if it were such that mathematical objects existed—then there would be certain further claims that would be true of it, and this consequence follows necessarily. It is important to note that C can be used by either contingentist fictionalists or necessitarian fictionalists. Contingentist fictionalists will interpret C as a counterfactual, and take the existence of mathematical objects to be possible, albeit not actual, whereas necessitarian fictionalists will interpret C as a counterpossible—as they hold that abstract objects necessarily do not exist. This is equivalent to the statement that it is not possible for abstract objects to exist; as such, when necessitarians use C, they are assuming an antecedent they take to be impossible. If the antecedent is impossible, then assuming it entails a contradiction. As such, necessitarians will need to adopt some non-classical logic in order to explain how conditionals with impossible antecedents can be true without entailing that all such conditionals are true. This is because, in classical logic, a contradiction entails the truth of all statements. I will consider only the contingentist view, suggesting later that this view is prima facie more plausible than the necessitarian alternative.

On the standard Lewisian analysis of counterfactuals, a counterfactual (A$\Box\!\!\Rightarrow$B) is true if and only if in the nearest possible worlds in which the antecedent obtains, the consequent also obtains (Lewis 1986). Fictionalists, however, presumably reject the ontology of possible worlds. In speaking of possible worlds, I do not intend to commit to their ontology; when I refer to them, I do so only in order to cash out features of our modal-talk. For present purposes, it suffices to point out that while there is no consensus on what the correct analysis of counterfactuals is, any plausible view grants that if A entails B (A$\models$B), then the counterfactual "if A had been true then B would have been true" (A$\Box\!\!\Rightarrow$B) is true. It turns out that this is all that is required for the counterfactual analysis to be true, because C is necessarily true due to the entailment relation between A and B. Our standard mathematical theories make reference to 3, and the antecedent assumes that our standard mathematical theories are true descriptions of reality, so the consequent simply makes explicit what is implicit in, or entailed by, the antecedent. It is necessary analogously to the way the counterfactual "If there were bachelors on the moon, then there would be unmarried men on the moon" is necessary. As such, the consequent obtains in every possible world in which the antecedent obtains. Balaguer's view holds that the correctness conditions of mathematical statements are contained in the counterfactual, and in this way it is capable of avoiding several of the objections that have been leveled against RAF. The correctness conditions for mathematical statements depend upon how mathematical objects *would have been* if they existed, rather than upon concrete sentence tokens that are about abstract objects.

Moreover, this kind of contingentism that could be paired with Balaguer's view is a totally different kind of contingentism from that which is supported by RAFs, and it is not a bad kind of contingentism. A contingentist of this type would say that a statement or proposition is contingent if it is actually true but it could be false, or vice versa, that it is actually false but could be true [**(-Φ & ◊Φ)** or **(Φ & ◊-Φ)]**. The counterfactual contingentist fictionalist (CCF) holds that while abstract objects do not actually exist, they could, i.e., the non-existence of numbers is not necessary. According to CCFs, our mathematical theories are contingent because the existence of mathematical objects is contingent, whereas the correctness conditions for mathematical statements are necessary. In contrast, the contingentism in RAF holds that the correctness conditions for mathematics are dependent upon the existence of sentence tokens and people.

According to CCFs, abstract objects do not exist in the actual world, but it is not necessary that they do not exist. CCFs arrive at the plausibility of this claim by conducting a thought experiment. They consider the sentence Ω: "Abstract objects do not exist but it is possible that they exist." (-Φ & ◊Φ) to see if there is a contradiction that would render the sentence impossible.[4] This statement does not entail a semantic or logical contradiction, so there is no incoherence arising from the meaning of the terms.

This leaves for consideration facts about physical-empirical reality that would render the contingentist thesis incoherent and absurd. The CCFs cannot grasp any such facts—there is nothing contradictory in the actual world being as it is, and Φ being possible (i.e., that numbers could exist)—so they cannot see any reason for believing that abstracta necessarily do not exist. According to CCFs, it is actually true that numbers do not exist, but it is not absurd to think of an alternative in which they could have existed.

V. Objectivity for Correctness Conditions Recovered

CCFs thus make a much less controversial claim than RAFs; they simply hold that mathematical objects do not exist, and that there is no necessity involved in this lack of existence. This version of fictionalism (CCF) is true just in case there is a possible world in which mathematical objects exist, but they do not exist in this world. RAF, in contrast, uses descriptive statements in order to capture correctness conditions and thus involves commitment to written or uttered theory-tokens. RAFs claim that certain sentences are true-in-the-story-of-mathematics because those sentences are members of a theory or are entailed by the sentences that constitute that theory. As such, RAFs claim that mathematical correctness is determined not by how the world *could be*, but by how it is—and this is a weakness for the referential account. Referential fictionalists must defend the claim that which mathematical statements are correct depends on the existence of tokens in the physical world.

By making the correctness conditions depend on the way in which certain abstract objects would have been, fictionalists are capable of capturing a higher degree of objectivity concerning the nature of mathematics, and of escaping worries about arbitrariness that are incurred when claiming that correctness conditions are determined by sentence tokens. Given that the counterfactual is about abstract objects of the kind that **our mathematical theories purport to be about**, it may seem that this brings in some degree of relativity or cultural arbitrariness, as the correctness conditions are not simply determined by the nature of abstract objects alone but partially by facts about our semantic intentions as well—i.e., what we intend to pick out as our story of mathematics. However, this feature of the counterfactual does not make the correctness conditions arbitrary so much as it specifies the range of abstract objects that can count as being the ones that set the correctness conditions for *our* story of mathematics. There could be a series of natural numbers that is very different from the one that we commonly think of when we are doing mathematics—one that lacks any number over 52, or in which there are numbers that do not have finitely many predecessors. If mathematical objects of the aforementioned kind exist, they are not the ones with which we are standardly concerned, and so they are not the ones that determine the correctness conditions of our mathematical theories.

To illustrate how CCF obtains a higher degree of objectivity for mathematical correctness, we can consider the objection that if there are no people, then no mathematical statements will be fictionally correct. Referential contingentist fictionalists must accept this conclusion. On the other hand, the counterfactual contingentist fictionalists say that the correctness conditions for a given branch of mathematics (e.g., set theory) are given by our conception of that branch of mathematics and its logical entailments. On their view, when we are trying to capture the correctness conditions for a particular story of mathematics, we assume the counterfactual supposition that objects of the *specified* kind exist, and this does not depend on there being semantic intentions—it holds that if abstract objects (of the intended type) existed, then certain mathematical sentences would be correct. This account is thus thoroughly conditional; it does not depend upon people existing for there to be truths about what counts as correct or incorrect within a sufficiently well-specified story. Thus, according to CCFs, there is a possible world *w* in which there are no people but which contains abstract objects of the kinds that our mathematical theories purport to be about and in which 3 is prime.

One further important feature of CCF is that fictionalists and Platonists will be in rough agreement about the correctness conditions of mathematical statements; all that they will disagree upon is the truth-value of the antecedent in C: "*If the abstract objects that our theories purport to be about existed, then 3 would be prime*". Fictionalists will say that the antecedent is false but nonetheless possible. Platonists will simply take the antecedent as a true description of the way the world is. Moreover, in both cases, C is necessary, thus preserving the objectivity of mathematical correctness. Mathematical correctness is belief-independent analogously to the way correctness about logical entailments is belief-independent. Once we have concerned ourselves with a particular conception of mathematical objects of a certain kind, e.g., natural numbers, the correctness of a mathematical claim does not depend on our beliefs. 3 is prime regardless of what a particular person says or thinks. Furthermore, even if mathematical objects do not exist, there are objectively correct statements of mathematics, and our standard mathematical theories will have objective correctness conditions.

VI. Conclusion

Szabó has argued that fictional correctness conditions of mathematical statements must be given by concrete sentence tokens, and that fictionalism thus entails an implausible dependence thesis. I have shown that fictionalists can and should avoid this conclusion. If fictionalists hold that mathematical objects exist contingently, and that the fictional correctness conditions for mathematical statements are given by a counterfactual of the form C: "*If the abstract objects that our theories purport to be about existed, then 3 would be prime"*, then they do not need to be committed to the existence of anything in order to claim that some statements are correct and others are incorrect. I have argued that the contingentism inherent in this view does not entail any deep theoretical problems and is relatively plausible. CCF holds that the non-existence of mathematical objects is contingent, but that mathematical correctness is nonetheless necessary and belief-independent, given a counterfactual supposition. This account of mathematical correctness conditions is thus non-metaphysical in the sense that fictional correctness does not depend on the existence of anything, e.g., people or numbers. In adopting this version of contingentism, fictionalists are capable of endorsing correctness conditions that are roughly analogous to the correctness conditions that Platonists will endorse, thus preserving the close semantic connection between fictionalism and Platonism.

Notes

[1] The kind of mathematical fictionalism I will argue for endorses the view that mathematical singular terms do not refer and therefore mathematical statements are false. Others that endorse a different view of vacuity hold that mathematical statements lack truth value.

[2] There is more than one mathematical "story" that deviates from our standard mathematics. I will use the truth predicate "*true-in-the-story-of-mathematics"* as meaning true in the story of what our mathematical theories purport to be about. For further reference see Balaguer 1998, p. 13.

[3] If instead of a counterfactual, a material conditional was used, then many mathematical statements that we take to be intuitively false would end up vacuously true; for instance, "If there are numbers, 2 is odd" is vacuously true if the antecedent is false.

[4] A similar argument can be found in Rosen 2002, p. 294.

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