# CONTRADICTION, ASSERTION AND 'FREGE'S POINT'

*By* GRAHAM PRIEST and RICHARD SYLVAN

THE liar sentence ('This sentence is not true') is both true and not true. The Russell set ({x: x ∉ x}) is both a member of itself and not a member of itself. These and many other contradictions we, the authors, are prepared to assert. Most people would disagree with us — and disagree fairly violently. But there is a minority of people who would refrain from disagreeing on the ground that we have said nothing to disagree with. Though we may have uttered words, no statement is made by them. Explicit contradictions cannot be used to make a statement.

This line, though hardly orthodox, has, of course, a distinguished pedigree. It was argued by Aristotle in *Metaphysics* Γ3,[1] and since then has resurfaced from time to time in the cancellation view of negation. (See Routley and Routley [5].) Most recently, it has been advocated by Laurence Goldstein [2], who has produced a novel argument for it. The purpose of this note is to refute the argument.[2] As we shall see, this is not difficult; the discussion, however, raises some points of independent interest.

Goldstein's argument concerns the following thesis ([2], p. 10):

> To assert a disjunction of disjunct propositions is to assert no disjunct, and to assert a [bi]conditional is to assert neither the antecedent nor consequent. (P. T. Geach [1] calls this Frege's point.)

We have inserted the word 'bi' here, since Goldstein obviously intends the point to apply to biconditionals as much as conditionals; indeed, when he applies the point it is to a biconditional. The attribution to Geach, though, is rather misleading. What Geach calls 'Frege's point' in the reference cited is the point that a thought has the same content asserted and unasserted. Still, the thesis, whatever its genesis, seems correct. (The following is obviously a fallacious reply: 'Zorn's Lemmas is true if the Axiom of Choice is true.' 'No, that isn't right: the Axiom of Choice is false.')

The argument itself is by reductio, and has three major premisses, the second of which is the above thesis:

(1) To assert a conjunction is to assert each conjunct.

[1] Though the arguments are not very good. See Łukaciewicz [3]. The arguments are also discussed in Priest and Routley [4], ch. 1, 'A Preliminary History of Paraconsistent and Dialetheic Approaches', section 5.2.

[2] Goldstein also briefly rehashes a number of older arguments, which are all variants on the theme that contradictions have no content. (Contradictions have no consequences, cannot be thought, cannot be understood etc.) We have dealt with this argument elsewhere. (Priest and Routley [4], ch. 5, 'The Philosophical Significance and Inevitability of Paraconsistency', section 3.1.) All the variants are false. We will not take up the matter again here.

(2) To assert a biconditional is not to assert either side of the biconditional.
(3) To assert $\alpha$ is to assert anything obviously logically equivalent to $\alpha$.

Suppose now that someone asserts $\alpha \wedge -\alpha$. The reductio goes as follows:

The person asserts both $\alpha$ and $-\alpha$ (by 1). But $\alpha \wedge -\alpha$ is obviously logically equivalent to $\alpha \leftrightarrow -\alpha$. Hence the person asserts $\alpha \leftrightarrow -\alpha$ (by 3); whence the person does not assert $\alpha$ or $-\alpha$ (by 2).

The argument might be criticized from various positions.[3] However, the first objection that we would bring is that $\alpha \wedge -\alpha$ and $\alpha \leftrightarrow -\alpha$ are not logically equivalent. Nor is this *ad hoc*. Virtually every account of the conditional which avoids the blunder of identifying it with material implication rejects this equivalence. Thus, relevant logics, many paraconsistent logics, Lewis/Stalnaker conditionals, and so on, all reject this equivalence. In fact, many of these would even reject a one-way implication between the two.

This reply is quite sufficient to refute the argument, but there is more to the matter than this. For a start, Goldstein's argument may be turned against those who accept material implication as the correct account of the conditional (who are, of course, not dialetheists). For such people (classical or intuitionist logicians) accept the equivalence of $\alpha \leftrightarrow (\alpha \rightarrow \alpha)$ and $\alpha$ as obvious, and, moreover, are prepared to assert $\alpha$ for various $\alpha$. Goldstein's argument would equally show this to be impossible.[4]

Whilst we are delighted to turn the tables, we will now (uncharacteristically) spring to the defence of our opponents. This we do in the interests of truth, justice, universal harmony, etc. For there is a problem with premiss (2). The only natural way to negate a sentence of the form 'to do this is to do that' is to say 'to do this is not to do that'; but this latter could also mean that doing this implies not doing that, which is quite different. (2) is therefore ambiguous. Using an obvious symbolism, and considering only one side of the biconditional (symmetry taking care of the other), it could mean either of:

(2a) $\vdash (\alpha \leftrightarrow \beta) \Rightarrow \nvdash \beta$
(2b) $-(\vdash (\alpha \leftrightarrow \beta) \Rightarrow \vdash \beta)$

Let us consider these in turn. (2a), whilst making the argument valid, is obviously false. Asserting $\alpha \leftrightarrow \beta$ does not preclude one from asserting $\beta$. Indeed, someone may quite consistently assert $\beta$

[3] For example, (1) might be criticized by a connexivist, who rejects the inference from $\alpha \wedge \beta$ to $\alpha$. (See Routley *et al.* [6], ch. 2, section 4.)

[4] Moreover, the disjunction version of (2) would seem to pose a problem for everyone, since all can agree that $\alpha \vee \alpha$ is obviously logically equivalent to $\alpha$.

and $\alpha \leftrightarrow \beta$ (if, for example, they assert also $\alpha$). The intent of (2) is that someone who asserts $\alpha \leftrightarrow \beta$ does not thereby assert $\beta$; they may, of course, assert $\beta$ independently. Thus, the content of (2) (and what Goldstein intends) is much more like (2b). This is not so obviously false. Moreover, it, together with (3), does lead to a contradiction. For if we can find any conditional, $\alpha \leftrightarrow \beta$, which is obviously equivalent one of its sides, $\beta$, then (3) gives:

$$\vdash \alpha \leftrightarrow \beta \Leftrightarrow \vdash \beta$$

Substituting equivalents in (2b) gives: $-(\vdash \beta \Rightarrow \vdash \beta)$, which is obviously false.

Given that only (2b) and (3) are involved in this argument essentially, it may be seen as a reductio of their conjunction. A simple way out is to reject (3), which is highly dubious. For example, 'The sun is shining' is logically equivalent to 'The sun is shining and (the sun is shining or all sub-atomic particles have integral spin)' ($\alpha \leftrightarrow \alpha \wedge (\alpha \vee \beta)$), and this may be obvious to a speaker; yet it seems implausible to say that someone who asserts the former asserts the latter: an auditor may understand the former assertion but be incapable of understanding the latter since they have never heard of particle physics. Yet this is too easy a solution. For a start, we may be able to find an $\alpha$ and a $\beta$ where the connection between $\alpha \leftrightarrow \beta$ is much tighter than obvious logical equivalence, and which do, therefore, make the same assertion.[5] Moreover, suppose we interpret 'assert' more liberally as 'be committed to' then (3) is true: people are committed to all the logical consequences of their commitments (or at least, all those that follow in the logic they accept). Moreover, (2b) seems equally correct on this understanding: it is not, in general, the case that if you are committed to $\alpha \leftrightarrow \beta$ you are committed to $\beta$. Hence the problem is still with us.

The solution is, in fact, that (2b) is false. To see this, observe that (2) suffers from a further ambiguity. It contains an implicit universal quantifier ('a biconditional'). And negation and a quantifier together always spell ambiguity. Consider, for example, 'All women are not stupid'. This can mean that no women are stupid. But equally (and contrary to received logicians' wisdom), it can mean that it is not the case that all women are stupid. (Try it: 'All women are stupid. No, all women are not stupid.') Thus (2) might mean either of:

$$\forall \alpha \forall \beta - (\vdash (\alpha \leftrightarrow \beta) \Rightarrow \vdash \beta)$$

$$- \forall \alpha \forall \beta (\vdash (\alpha \leftrightarrow \beta) \Rightarrow \vdash \beta)$$

(where the quantifiers may be thought of as substitutional). The first of these is (2b) and leads to trouble. The second does not.

[5] *A fortiori* for the disjunction version of the argument. It is very plausible to think that $\alpha$ makes the same assertion as $\alpha \vee \alpha$.

And this, we claim, is the correct content of (2). In general, and normally, to assert a conditional is not to assert its consequent; in particular cases, however, it may be.[6] This is, we think, a main lesson of Goldstein's argument.

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## References

[1] P. Geach, 'Assertion', *Philosophical Review* 74 (1965), 449–65. Reprinted as 8.2 in Geach's *Logic Matters*, (Blackwell, 1972).

[2] L. Goldstein, 'A Problem for the Dialetheist', *Bulletin of the Section of Logic, Polish Academy of Sciences* 15.1 (1986), 10–14.

[3] J. Łukaciewicz, 'On the Principle of Contradiction in Aristotle', *Review of Metaphysics* 24 (1971), 485–509.

[4] G. Priest and R. Routley, *On Paraconsistency*, Research Report #13, Research School of Social Sciences, Australian National University, 1983. Reprinted in G. Priest *et al.* (eds.) *Paraconsistent Logics* (Frankfurt: Philosophia Verlag, 1988).

[5] R. Routley and V. Routley, 'Negation and Contradiction', *Revista Columbiana de Mathematicas* 19 (1985), 201–31.

[6] R. Routley *et al.*, *Relevant Logics and their Rivals* (Ridgeview, 1982).

[6] Similarly for disjunctions. Normally, to assert $\alpha \vee \beta$ is not to assert $\alpha$ or $\beta$. But for certain instances of $\alpha \vee \beta$ (e.g. $\alpha \vee \alpha$) it may be.

# SORENSEN'S SORITES

*By* ROBERT DEAS

ROY SORENSEN has produced an ingenious argument (ANALYSIS 45.3, June 1985, pp. 134–7) in an attempt to show that the predicate 'vague' is itself vague, and generates paradoxes in the same manner as its instances. I do not think that he succeeds. He has, I shall argue, merely disguised an already familiar sorites argument, and the vagueness he attributes to 'vague' is rightfully that of quite another predicate.

He begins by presenting a series of novel predicates, '1-small', '2-small', etc., the nth of which is defined as applying to all and only integers which are either less than n, or small. Now since 'small' itself is acknowledged to be vague, it is obviously true that '1-small' is vague, because both it and 'small' apply to 0, the only integer less than 1, and in virtue of the definition both apply equally to all other integers. Furthermore, Sorensen argues, it is clearly true that for any given integer n, if 'n-small' is vague then 'n+1-small' is vague. This, of course, gives us the premisses of a standard sorites