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Labour Theory of Value and Marginal Utilities[1]

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I. INTRODUCTION

In a recent article [4] I commented briefly on the relationship between the Marxist labour theory of value and the theory of marginal utilities. I quote from the article:

> "The Marxist labor theory of value has been the object of attacks particularly from the point of view of "marginal utility theory" or "subjective theory of value", which has been a main component of non-Marxist mathematical economics. Marxists have usually rejected this whole theory and all concepts and mathematical arguments introduced in connection with it, as if acceptance of it, or elements of it, would necessarily imply a rejection of the labor theory of value. However, this is not so. For goods which can be reproduced on any scale (i.e. such goods as have been the center of interest of Marxian value theory) it is very easy to demonstrate that a complete model still leaves prices determined by the labor theory of value even if one accepts the marginal utility theory of consumers' behavior. In my opinion, such a combination may give a very precise meaning to the Marxian thesis that the value of a product is determined by its labor content, *provided it has a use value.*"

Since then I have received several letters asking me to substantiate the contention contained in the paragraph quoted above. The following is an attempt to do so.

A suitable starting point for such an attempt is the by now familiar input-output analysis. The similarities and connections between the price theory of input-output models and the labour theory of value have been pointed out e.g. in Burgess Cameron [1], Michio Morishima and Francis Seton [5] (see also [3]), Jacob T. Schwartz [6], and by the present writer in [2]. On some points the presentation which is to be given here is influenced by Schwartz' analysis.

[1] The present article does not deal explicitly with problems of economic planning. Nevertheless, we think the article will be of special interest to economists studying and working with the theory of centrally planned economies. *The Editor.*

We shall in the following restrict ourselves to considering static models describing what is called in Marxian theory simple reproduction. I think this suffices for an elucidation of *some* of the basic problems of price and value theory.

II. THE PARTIAL PRICE MODEL

Let us first consider a bare input-output model where no utilities and demand functions enter. Production is considered as a flow, i.e. we consider output volume per unit of time. There are n commodities denoted by $A_1, A_2 \ldots A_n$. Other notations are as follows:

a_{ji} = the amount of A_j used up in the production of one unit of A_i. It includes wear and tear on equipment.
β_{ji} = the physical *stock* of A_j which is "tied up" in the production of A_i, per unit of A_i per unit of time.
γ_i = the amount of (homogeneous) labour used up in the production of one unit of A_i.

We assume that a_{ji}, β_{ji} and γ_i for $i, j = 1 \ldots n$ are technologically fixed coefficients. This is a standard assumption in input-output theory and it also conforms with the assumptions underlying the simpler expositions of the Marxian labour theory of value. In particular the coefficients are independent of the scale of production. (Cf. the expression "goods which can be reproduced on any scale" in the above quotation.)

P_i = the price of A_i;
w = the wage rate (in money terms);
ϱ = the rate of profit.

Assuming homogeneous labour, we have only one wage rate. Furthermore, assuming competition to prevail, and studying only equilibrium conditions and not fluctuations, the rate of profit will be the same in all branches. The price of A_i will then consist of three components: First, expenses on material inputs per unit of A_i which are $\Sigma_j P_j a_{ji}$; second, wage expenses which are $w\gamma_i$; and third, normal profits on the capital stock. The value of the physical capital stock tied up in the production of A_i per unit of product (per unit of time), is in terms of prices, $\Sigma_j P_j \beta_{ji}$; the amount of profit on this per unit of output of A_i is accordingly $\varrho \Sigma_j P_j \beta_{ji}$. Furthermore we shall assume that the producers must also hold an additional capital, as advances to labour, which is proportional to the wage sum per period. For simplicity we shall assume that the factor of proportionality equals unity.[1] Then an additional profit component equal to $\varrho w\gamma_i$ must also be included in the price.

From this we obtain the following system of equations for the price formation:

$$(1) \qquad P_i = \sum_{j=1}^{n} P_j a_{ji} + w\gamma_i + \varrho(w\gamma_i + \sum_{j=1}^{n} P_j \beta_{ji}) \qquad (i=1\ldots n).$$

This is a system of n linear equations in the unknown prices $P_1 \ldots P_n$, if we, for the moment, consider w and ϱ as given magnitudes. Under plausible assumptions about the coefficients which enter the formula, this will determine the prices. (Compare e. g. Jacob T. Schwartz [6], lecture 3.) We may therefore draw the following conclusion: If the wage rate and the profit rate are given, prices are determined completely by the coefficients a_{ji}, β_{ji} and γ_i which reflect the technology of production. If marginal utilities should affect the prices, it must therefore be through the wage rate and profit rate in an extended model where these magnitudes are no longer considered as determined by factors "outside the model".

The role of the wage rate is, however, a restricted one. The wage rate (in monetary terms) does not influence the relative prices. In fact, we could divide through the equations in (1) to get

$$(1^*) \qquad \frac{P_i}{w} = \sum_{j=1}^{n} \frac{P_j}{w} a_{ji} + \gamma_i + \varrho(\gamma_i + \sum_{j=1}^{n} \frac{P_j}{w} \beta_{ji}).$$

Once the rate of profit ϱ is given, this would determine all magnitudes $\frac{P_1}{w} \ldots \frac{P_n}{w}$, i.e. all prices relative to the wage rate. Any change in w would cause only a proportional change in $P_1 \ldots P_n$, and the relative prices P_i/P_j would remain independent of w.[2] Therefore the interesting factor through which prices could depend on marginal utilities, is the rate of profit. This possible dependence of prices upon marginal utilities is a rather weak and indirect one. If we consider the space of all relative prices, shifts in marginal utility curves can at most generate a one-dimensional variation in this space, and the price of a commodity will

[1] If the factor is the same for all sectors, this is only a matter of choosing the length of the time unit.

[2] Instead of assuming w to be given in monetary terms, one could introduce a commodity, say No. 1, which serves as money. Then we should have $P_1 = 1$, and w would be one of the "prices" to be determined by the equations in (1). This would correspond to some of Marx' considerations on money. The problem of how to determine the nominal level of those variables $P_1 \ldots P_n$ and w which are expressed in monetary terms, is however not important for the following analysis.

not change in any obvious and simple way as a result of a shift in the marginal utility curve of that particular commodity.[3]

In a special case the relative prices would be independent of the rate of profit ϱ. This is the case of "equal organic composition of capital". This assumption can be interpreted and expressed in different ways. Let us first express it by requiring that the value in money terms of physical capital in proportion to the total capital held should be the same in all sectors:

$$\sum_{j=1}^{n} P_j \beta_{ji} = \lambda(w\gamma_i + \sum_{j=1}^{n} P_j \beta_{ji}) \qquad (i = 1 \ldots n) \tag{2}$$

in which λ is independent of i. In this case (1) reduces to

$$P_i = \sum_{j=1}^{n} P_j a_{ji} + (1 + \frac{\varrho}{1-\lambda})\, w\gamma_i \qquad (i = 1 \ldots n). \tag{3}$$

It is well known from Marxian economics that prices should in the case of equal organic composition of capital be proportional to the labour values of the commodities. Let us now check this.

The labour values (expressed in units of labour) will be denoted by $\underset{\sim}{Q}_1, \underset{\sim}{Q}_2, \ldots, \underset{\sim}{Q}_n$.

The amount of labour used up directly in producing one unit of A_i is γ_i. The amount of labour used up indirectly through the inputs from other sectors is $\Sigma_j \underset{\sim}{Q}_j a_{ji}$. (Remember that wear and tear are included in a_{ji}.) The total amount of labour contained in one unit of A_i, i.e. the labour value of A_i expressed in units of labour, is therefore

$$\underset{\sim}{Q}_i = \gamma_i + \sum_{j=1}^{n} \underset{\sim}{Q}_j a_{ji} \qquad (i = 1 \ldots n). \tag{4}$$

On the usual assumptions about the input-output coefficients, this system has a unique (positive) solution.[4]

It is now easy to demonstrate that the prices $P_1 \ldots P_n$ which solve the system (3) will be proportional to the labour values $\underset{\sim}{Q}_1 \ldots \underset{\sim}{Q}_n$ which are determined by (4). For this purpose, let us write

$$P_i = \varphi \underset{\sim}{Q}_i \qquad (i = 1 \ldots n) \tag{5}$$

where φ is a factor of proportionality. Let us insert this in (3), to yield

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[3] See Jacob T. Schwartz [6], pp. 188 and 196–97.

[4] For interpreting the concept of labour values it may be of some interest to observe that the equation system (4) is the same as (1*) (with $\frac{P_1}{w} \ldots \frac{P_n}{w}$ as variables), if $\varrho = 0$, i.e. if there is no profit on capital included in price.

$$\varphi Q_i = \varphi \sum_{j=1}^{n} Q_j\, a_{ji} + (1 + \frac{\varrho}{1-\lambda})\, w\gamma_i \qquad (i = 1\ldots n),$$

or

$$\varphi(Q_i - \sum_{j=1}^{n} Q_j\, a_{ji}) = (1 + \frac{\varrho}{1-\lambda})\, w\gamma_i \qquad (i = 1\ldots n).$$

Since we have, from (4), $Q_i - \Sigma_j\, Q_j\, a_{ji} = \gamma_i$, we see that $P_1 \ldots P_n$ in (5) satisfy (3) if the factor of proportionality φ is

$$\varphi = (1 + \frac{\varrho}{1-\lambda})\, w. \tag{6}$$

By (2) we expressed the assumption of equal organic composition of capital in money terms by means of the prices $P_1 \ldots P_n$. We might also express the assumption by

$$\sum_{j=1}^{n} Q_j\, \beta_{ji} = \lambda'(\sum_{j=1}^{n} Q_j \beta_{ji} + \gamma_i) \qquad (i = 1\ldots n). \tag{7}$$

The condition (7) is equivalent to (2), both being necessary and sufficient for the prices as obtained from (1) to be proportional to the labour values as obtained from (4). The measures for the composition of capital in (2) and (7) are, however, not necessarily equal, i.e. λ' is generally not equal to λ.[5]

By means of the concept of organic composition of capital, we can develop a simple correspondence between the rate of surplus value and the profit rate. In monetary units the profit in branch i per unit of output is

$$\pi_i = P_i - \sum_{j=1}^{n} P_j\, a_{ji} - w\gamma_i \qquad (i = 1\ldots n). \tag{8}$$

Denoting now by λ_i the organic composition of capital in branch i, we have corresponding to (2)

$$\sum_{j=1}^{n} P_j\, \beta_{ji} = \lambda_i\, (w\gamma_i + \sum_{j=1}^{n} P_j\, \beta_{ji}) \qquad (i = 1\ldots n). \tag{9}$$

Using (1), (8) and (9), we obtain the following relationship between

[5] In chapter IV we shall introduce the concept of value of labour power, denoted by Q_L. Using that, we could instead of (7) write:

$$\sum_{j=1}^{n} Q_j\, \beta_{ji} = \lambda''(\sum_{j=1}^{n} Q_j\, \beta_{ji} + Q_L\, \gamma_i) \qquad (i = 1\ldots n).$$

With the formula for Q_L given in chapter IV, this would again be equivalent to (2), and we would have $\lambda'' = \lambda$.

profits as a proportion of the wage sum, the rate of profit and the organic composition of capital:[6]

$$\mu_i = \frac{\pi_i}{w\gamma_i} = \frac{\varrho}{1-\lambda_i} \qquad (i = 1 \ldots n). \tag{10}$$

It may finally be of some interest to consider another case. Let us assume that $\beta_{ji} = a_{ji}$, i.e. that the physical capital tied up in a sector is equal to the consumption in one period of each input. I think some assumptions occasionally made by Marx in *Capital* can be expressed in this way. Then (1) can be written as

$$P_i = (1 + \varrho)\left(\sum_{j=1}^{n} P_j \beta_{ji} + w\gamma_i\right) \qquad (i = 1 \ldots n). \tag{11}$$

We now let the organic composition of capital vary as between sectors, as in (9). Then

$$P_i = (1 + \varrho)\frac{w\gamma_i}{1-\lambda_i} \qquad (i = 1 \ldots n). \tag{12}$$

It is seen that there is in this case a simple relationship between the price of a commodity and the organic composition of capital in the branch, the price being (for a given γ_i) higher the higher the organic composition of capital. This may be of interest in interpreting some propositions in Marx' *Capital*. However, it should be remembered that λ_i is itself calculated by using the price structure, and is therefore not a purely technical characteristic.

III. A REMARK ON MACRO-ECONOMIC RELATIONSHIPS

Before we extend the model presented so far, let us consider briefly some implied macro relationships. We must then first introduce a symbol for the output of each branch: Let it be X_i for branch No. i.

Total wage income will then be

$$W = w\sum_{i=1}^{n} \gamma_i X_i = wN, \tag{13}$$

where N is now total labour input.

Total profits will be

$$\Pi = \sum_{i=1}^{n} \left(P_i - \sum_{j=1}^{n} P_j a_{ji} - w\gamma_i\right)X_i. \tag{14}$$

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[6] Formula (10) corresponds to the formula given on p. 68 of Paul M. Sweezy [7].

From equation (1) it appears that this can be written as

$$\Pi = \varrho \sum_{i=1}^{n} (w\gamma_i + \sum_{j=1}^{n} P_j \beta_{ji}) X_i = \varrho K, \tag{15}$$

in which K now is total capital held in monetary terms. Using (9) we can further write this as

$$\Pi = \varrho \sum_{i=1}^{n} \frac{w\gamma_i X_i}{1-\lambda_i}. \tag{16}$$

If we define a certain average organic composition of capital $\bar{\lambda}$, (16) can be written as

$$\Pi = \varrho \frac{wN}{1-\bar{\lambda}} = \varrho \frac{W}{1-\bar{\lambda}}. \tag{17}$$

The precise nature of $\bar{\lambda}$ is given by the relation

$$\frac{1}{1-\bar{\lambda}} = \frac{\sum_{i=1}^{n} \frac{1}{1-\lambda_i} \gamma_i X_i}{\sum_{i=1}^{n} \gamma_i X_i}, \tag{18}$$

i.e. the complement of $\bar{\lambda}$ is a weighted harmonic mean of the complements of the organic composition of capital in the different branches.

From (17) we have

$$\mu = \frac{\Pi}{W} = \frac{\varrho}{1-\bar{\lambda}}. \tag{19}$$

This corresponds completely with equation (10) for the individual branches. Comparing (19) and (10), we see that the ratio between profits and wages in a branch is higher than the same ratio for the economy as a whole when the organic composition of capital in that branch is higher than the average, and lower when the organic composition of capital is lower than the average. If the organic composition of capital is the same in all branches, the ratios between profits and wages in the different branches ($\mu_1 \ldots \mu_n$) will all be equal and then, of course, also equal to the same ratio for the economy as a whole. From this it appears that the special case of equal organic composition of capital is both convenient and relevant when one wants to study problems of a macro-economic type, and in particular the distribution between profits and wages which is at the center of Marx' analysis. As is well known, many students of Marx interpret Volume I of *Capital*, in which this special assumption is often made, as an attempt mainly at clarifying problems which in our present terminology could be classified as being of a macro-economic type.

If the organic composition of capital is equal in all branches, we have seen that prices are proportional to values. The μ introduced above, which is then $\mu = \mu_1 = \ldots = \mu_n$, can then be interpreted as the rate of surplus value. This will be further clarified in the next chapter.

IV. AN EXTENDED MODEL WITH MARGINAL UTILITY FUNCTIONS FOR THE CAPITALISTS AND FIXED CONSUMPTION REQUIREMENTS FOR THE WORKERS

We have now satisfied ourselves that the price equations (1) embed the main assumptions of Marxian price theory and also bring out many of its conclusions. We shall next see how these equations can be combined with other equations so as to form more complete models. In particular we shall see how they can be combined with marginal utility functions without running into logical contradictions.

We shall first consider a model in which there are defined some minimum quantities of the different commodities which are necessary to "reproduce the labour power", and assume that wages are just sufficient to buy these quantities.[7] In this sense the workers receive wages corresponding to the value of the labour power. Capitalists, however, enjoy a higher consumption per head than the necessary minimum. They are therefore in a position to weigh against each other marginal quantities of the different commodities, and we shall now assume that they do this so as to maximize a utility function.

Let the quantity of the j'th commodity which is necessary per unit of labour, be η_j. The wage rate must then be

$$w = \sum_{j=1}^{n} P_j \eta_j. \tag{20}$$

The consumption of each commodity by the workers is

$$C_i^w = \eta_i N = \eta_i \sum_{j=1}^{n} \gamma_j X_j \qquad (i = 1 \ldots n). \tag{21}$$

For the capitalists we shall for convenience proceed as if there be only

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[7] In Marx' analysis this "necessary consumption" is, of course, not determined simply by physical or biological necessity. It is changing in response to changing technical conditions, social conditions, class struggle on the distribution of incomes etc. But these are slow and gradual changes, and cannot be subjected to analysis in such comparatively simple terms as other aspects of the mechanism which determine the prices.

one representative capitalist who consumes the quantities $C_1^c \ldots C_n^c$ [8]. These quantities are determined by the maximization of a utility function

$$U^c(C_1^c \ldots C_n^c) \tag{22}$$

subject to a budget constraint

$$\sum_{i=1}^{n} P_i C_i^c = \Pi . \tag{23}$$

Defining the marginal utilities by

$$u_i^c(C_1^c \ldots C_n^c) = \frac{\partial U^c(C_1^c \ldots C_n^c)}{\partial C_i^c} \qquad (i = 1 \ldots n), \tag{24}$$

we have the following maximizing conditions:

$$\frac{u_1^c(C_1^c \ldots C_n^c)}{P_1} = \ldots = \frac{u_n^c(C_1^c \ldots C_n^c)}{P_n}. \tag{25}$$

These together with the budget equation (23) determine the consumption demand of the capitalist as functions of the profit sum and the prices.

We are now in a position to collect all equations of the model:

$$\text{(a)} \qquad P_i = \sum_{j=1}^{n} P_j a_{ji} + w\gamma_i + \varrho(w\gamma_i + \sum_{j=1}^{n} P_j \beta_{ji}) \qquad (i = 1 \ldots n)$$

$$\text{(b)} \qquad w = \sum_{j=1}^{n} P_j \eta_j$$

$$\text{(c)} \qquad C_i^w = \eta_i \sum_{j=1}^{n} \gamma_j X_j \qquad (i = 1 \ldots n)$$

$$\text{(d)} \qquad \frac{u_1^c(C_1^c \ldots C_n^c)}{P_1} = \ldots = \frac{u_n^c(C_1^c \ldots C_n^c)}{P_n}$$

$$\text{(e)} \qquad \pi = \sum_{i=1}^{n} (P_i - \sum_{j=1}^{n} P_j a_{ji} - w\gamma_i) X_i \; [= \sum_{i=1}^{n} P_i C_i^c]$$

$$\text{(f)} \qquad X_i = \sum_{j=1}^{n} a_{ij} X_j + C_i^w + C_i^c \qquad (i = 1 \ldots n).$$

All these equations are introduced previously, except for the last set (f). These are the usual input-output equations with $C_i^w + C_i^c$ as the "final demand column", which is, however, endogeneous in the present model.

One might ask whether the budget equation (23) for the capitalists should be introduced in the model as an independent equation. This

[8] It would only require some heavier notational equipment, but no change in reasoning, to develop the model with a number of capitalists who gain profits in proportion to their shares in the total capital in formula (15).

equation is however already implied by the equations in (a–f). This can be seen by multiplying first (f) by P_i and summing over i. This yields

$$\sum_{i=1}^{n} P_i X_i - \sum_{i=1}^{n} \sum_{j=1}^{n} P_i a_{ij} X_j - \sum_{i=1}^{n} P_i C_i^w = \sum_{i=1}^{n} P_i C_i^c.$$

Interchanging here i and j in the double sum and using (b) and (c), we get

$$\sum_{i=1}^{n} (P_i - \sum_{j=1}^{n} P_j a_{ji} - w\gamma_i) X_i = \sum_{i=1}^{n} P_i C_i^c.$$

By the first equality in (e) this is now seen to imply (23). We have, therefore, only indicated this equation in a bracket after (e).

In the model (a–f) there are then $4n + 1$ equations and $4n + 3$ variables: $P_1 \ldots P_n$, $X_1 \ldots X_n$, $C_1^w \ldots C_n^w$, $C_1^c \ldots C_n^c$, ϱ, Π and w. Apparently there are then two degrees of freedom in the model. However, as explained earlier, the variable w does not play any interesting role in the model. We could divide through all equations (a), (b) and (e) by w and multiply by w in (d), thereby obtaining a system with only $4n + 2$ variables $P_1/w \ldots P_n/w$, $X_1 \ldots X_n$, $C_1^w \ldots C_n^w$, $C_1^c \ldots C_n^c$, ϱ, Π/w. Then there would be one degree of freedom left.[9] The reason for this is that we have not introduced *any* condition which determines so to speak the scale of operation of the economy – the level at which the "constant reproduction" is taking place. This degree of freedom could be eliminated for instance by assuming a given total amount of physical capital somehow measured. The point is not important for our main problem, and we shall not go deeper into it.

The most important property of the system (a–f) in the present context is that the set of equations (a) and (b) form a determinate subset of equations except for the fact already pointed out that there is an arbitrary factor of proportionality in the set of variables $P_1, \ldots, P_n, w$. This means that also the rate of profit ϱ is determined by this subset of equations.

The way in which ϱ is determined may be clarified by inserting for w in (a) from (b). This yields the following set of equations in $P_1, \ldots, P_n$ and ϱ:

$$P_i - \sum_{j=1}^{n} P_j \left[a_{ji} + \eta_j \gamma_i + \varrho(\eta_j \gamma_i + \beta_{ji})\right] = 0. \tag{26}$$

This system is linear and homogeneous in the variables $P_1 \ldots P_n$. It has a solution with non-zero prices if and only if its determinant equals zero:

[9] The same thing could, of course, be obtained by dividing by any of the variables $P_1 \ldots P_n$ and Π instead of w.

(27) $$\begin{vmatrix} 1-g_{11}-\varrho h_{11} & \dots\dots & -g_{n1}-\varrho h_{n1} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ -g_{1n}-\varrho h_{1n} & \dots\dots & 1-g_{nn}-\varrho h_{nn} \end{vmatrix} = 0$$

in which

(28) $$g_{ji} = a_{ji} + \eta_j \gamma_i \quad \text{and} \quad h_{ji} = \eta_j \gamma_i + \beta_{ji}.$$

This condition is an equation in the unknown ϱ. Mathematically it will in general be satisfied by a set of different solutions for ϱ, but one may reasonably conjecture that only one of these will be economically meaningful.[10]

With the assumptions introduced by (20–21) it is possible to define the value of the labour power which is used in production. Per unit of labour expended the "inputs for reproducing the labour power" are $\eta_1 \dots \eta_n$ units of the different commodities. The total value used to "reproduce" one unit of labour, which is the definition of the labour value of the labour power, is accordingly

(29) $$\underset{\sim}{Q}_L = \sum_{j=1}^{n} \underset{\sim}{Q}_j \eta_j$$

in which $\underset{\sim}{Q}_1 \dots \underset{\sim}{Q}_n$ are determined by (4).

In order to compare our presentation with Marx', it is of some interest to consider the definition (29) more closely in the case of equal organic composition of capital, which is, as we have already observed, a particularly convenient case for macro-economic interpretations of the model. In that case there is a simple correspondence (5) with φ determined by (6) between labour values and prices. Expressing the values $\underset{\sim}{Q}_1 \dots \underset{\sim}{Q}_n$ in terms of prices and the wage rate by means of (5) and inserting in (29), we obtain

$$\underset{\sim}{Q}_L = \frac{\sum_{j=1}^{n} P_j \eta_j}{(1 + \frac{\varrho}{1-\lambda}) w}.$$

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[10] Compare Morishima and Seton [5], p. 207, where a mathematically rather similar system is investigated.

The condition (27) can be rewritten in another form which allows a formal interpretation. First write (27) in obvious matrix notations as

(27*) $$|(I-g') - \varrho h'| = 0.$$

Assuming that $(I-g')$ is non-singular, we can write this condition as

(27**) $$|(I-g')^{-1}h' - \frac{1}{\varrho}| = 0.$$

From this form it appears that $1/\varrho$ is a latent root of the matrix $(I-g')^{-1} h'$.

However, using (20) or (b), this reduces to

$$Q_L = \frac{1}{1 + \frac{\varrho}{1-\lambda}}. \tag{30}$$

Using furthermore the relationship (19) between the rate of surplus value and the profit rate, and using λ instead of $\bar{\lambda}$ since the organic composition of capital is assumed to be the same in all branches, we have

$$Q_L = \frac{1}{1+\mu}. \tag{31}$$

It may perhaps be more interesting to solve this equation for μ to yield

$$\mu = \frac{1 - Q_L}{Q_L}. \tag{32}$$

In this form the equation says that the rate of surplus value is the ratio between the value created by one unit of labour, i.e. unity, in excess of the value of the labour power itself, and this latter value; or, in Marxian terms, the rate of surplus value is equal to the ratio of surplus labour to necessary labour.

After this digression, we return to the study of the whole model (a–f). We have seen that the profit rate ϱ and all proportions

$$P_1 : P_2 : \dots : P_n : w$$

are determined by the subset (a–b) of equations in the model. This means that the marginal utilities which enter the model through (d), do not at all influence prices. What then is their role? Their role is to determine the *quantities* of the various commodities consumed by the capitalists. Thereby they influence the extent and composition of production. They will furthermore, through (c), influence the total labour input $\Sigma_j \gamma_j X_j$ and the scale of production for the consumption of the workers.

Briefly stated: Prices (including the wage rate) are – except for an arbitrary factor of proportionality – determined by technological conditions as expressed by a_{ji}, γ_i, β_{ji} and the consumption requirements for the "reproduction of the labour power" (η_i). The marginal utility functions interact with the prices thus given only in determining the *quantities* to be produced and consumed of the different commodities.

The above is of course only a description of an equilibrium situation. As such I think our results conform very well with some statements by Karl Marx in Volume III, Chapter X of *Capital*, where he first discusses the price formation and then adds that "for a commodity to be sold at its market-value, i.e. proportionally to the necessary social labour contained in it, the total quantity of social labour used in producing the total mass of this commodity must correspond to the quantity of the social want for it, i.e., the effective social want." Other quotations from Marx in the same direction could be given.[11]

For a commodity to be produced at all ($X_i > 0$) and obtain a price as determined by (a–b), it is necessary: (A) either that it is a necessity for the workers ($\eta_i > 0$), or (B) that it for some quantity C_i^c consumed by the capitalists has a sufficiently high marginal utility u_i^c to allow the fulfilment of (d), or (C) that it directly or indirectly through the a-coefficients enters into the production of one or more commodities which satisfy (A) or (B) or both. In this sense we can say that the price of a commodity in the model (a–f) is determined as said by the equations (a–b) *provided that it has a use value.*[12]

V. REMARKS ON A MODEL WITH MARGINAL UTILITY FUNCTIONS BOTH FOR CAPITALISTS AND WORKERS

In the model in chapter IV we employed the assumption of fixed consumption requirements for the workers. We might, however, construct a model in which also the consumption of the workers is determined by marginal utilities. The model of chapter IV may in fact be considered as a limiting case of such a model.

Since the full exposition of such a generalized model will entail many repetitions of what is already explained in previous sections of this article, we shall here only give some brief suggestions.

For the construction of the generalized model we could interpret N (see ch. III) simply as the number of workers employed. We must then introduce a utility function for each of the workers; for simplicity they could be assumed to be equal.

Per period of time each worker would receive a wage w, and he would determine his consumption pattern so as to maximize his utility function subject to the budget constraint. Instead of being determined by such coefficients as $\eta_1 \dots \eta_n$ in (c), the variables $C_1^w \dots C_n^w$ would then be determined by equations formally similar to (d).

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[11] See Paul M. Sweezy [7], pp. 47–52.

[12] If (B) is not satisfied for a certain commodity, which, however, satisfies (A) or (C), the corresponding ratio should be deleted from (d) and replaced by the condition $C_i^c = 0$.

The main problem which would occur in such a model is that we would have no obvious way of replacing equation (b) in the model (a–f), which together with (a) determined the distribution of income between workers and capitalists as reflected in the rate of profit ϱ. Several ways are open to solve the problem:

(i) One may simply consider the rate of profit ϱ as determined by conditions outside our model, by struggle between labour and capital over distribution shares.

(ii) One could introduce the supply of labour in the model and use that in one way or other to determine the distribution between labour and capital, and thereby ϱ.

(iii) The solution which would be nearest to the model in chapter IV would be to prescribe a certain "necessary" utility level for the workers. Then the introduction of utility functions for the workers would simply change the model from assuming a minimum requirement for each commodity separately to assuming the existens of some necessary minimum satisfaction of wants which could, however, be achieved by different *compositions* of consumption.

Whichever of these solutions is chosen, all our conclusions from the partial price model in chapter II would hold. If (i) is chosen, also all conclusions of chapter IV would hold. If (ii) or (iii) is chosen, prices would no longer be determined quite independently of marginal utilities. However, as explained already in chapter II, the influence of marginal utilities upon prices would be the very indirect one through the rate of profit ϱ. If the marginal utility function (workers' or capitalists') for a commodity should shift, the main effect would be that the quantity consumed would change. The price would only change to the extent that ϱ would change; and if so, prices of other goods would also change.

As explained in chapter II, the relative prices would remain constant even if ϱ changes, if the organic composition of capital is the same in all branches.

The conclusions of this chapter and chapter IV as regards the effects of utilities upon prices and quantities, may, in terms of demand and supply, be considered as a multi-commodity generalization of the case of a horizontal supply curve and a downward sloping demand curve.

VI. CONCLUDING REMARKS

The present article has only been concerned with the logical question whether marginal utility theory and the labour theory of value do necessarily contradict each other or not. I shall not attempt to discuss and evaluate the realism of the models. Nor shall I discuss the usefulness of going via value theory to price theory as is done by Marx rather than restricting oneself to talking only about prices. Some arguments on this point are, however, given by Paul M. Sweezy [7], pp. 128–130.

In considering the conclusions reached in the previous sections, one should remember that we have studied only equilibrium positions. In a dynamic study of the movements of prices, demand functions would play a more important role. Also in the case of monopoly price formation the demand side would be important. Both the points mentioned here were clearly recognized by Marx, cf. Paul M. Sweezy [7], pp. 47-52 and 54–55.

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