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A NOTE ON "AGGREGATION IN LEONTIEF MATRICES AND THE LABOUR THEORY OF VALUE"

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THE INTENTION of this note is to add to Morishima's and Seton's paper[1] a corrollary regarding unit labour values; this requires some modification in the original notations used.

Assume that there is defined a physical measure for the product of each sector and let the quantity produced by sector i be X_i. Furthermore, let P_i be the price per unit of product from sector i and Q_i be the value per unit of product from sector i. We then have

$$W = \hat{X}P \qquad \text{and} \qquad \omega = \hat{X}Q$$

where $\hat{X}$ is the diagonal matrix of the X_i's.

Let X_{ij} be the input from sector i to sector j. The input-output coefficients in physical terms are then $\alpha_{ij} = X_{ij}/X_j$ and their matrix is related to the allocation matrix c by

$$\alpha = \hat{X}c\hat{X}^{-1} .$$

Let v_j serve as a measure of the physical labour input in sector j. We introduce the labour input coefficient γ_j by

$$v = \hat{\gamma}X .$$

The pattern of consumption by the workers in sector j is given by

$$s_{ij} = \eta_{ij}v_j \qquad \text{or} \qquad s = \eta\hat{v} ,$$

where s_{ij} is the quantity of goods produced in sector i and consumed by workers employed in sector j and the η_{ij} are considered as constants. Then

$$d = \hat{X}^{-1}s = \hat{X}^{-1}\eta\hat{\gamma}\hat{X} .$$

Our intention now is to demonstrate that *the values per unit of output of the various sectors defined by the formula of Morishima and Seton depend solely on the physical coefficients α, γ, and η introduced above.* In particular the solution is independent of the scale of output of the various sectors. The significance of this is that the values which can be computed by means of Morishima's and Seton's system on the basis of one constellation of the economy remain constant as long as the production structure and the consumption pattern of the workers remain constant.

Morishima's and Seton's formula reads

$$[(I - c') - \varrho_0 d']\omega = 0$$

[1] "Aggregation in Leontief Matrices and the Labour Theory of Value," this issue.

where ϱ_0 is determined by the condition that the determinant shall vanish. We here first introduce $\hat{X}Q$ for ω in order to get values per unit of output. Substituting further $\hat{X}^{-1}\alpha\hat{X}$ for c and the expression given above for d, we get

$$[(I - \hat{X}\alpha'\hat{X}^{-1}) - \varrho_0\hat{X}\hat{\gamma}\eta'\hat{X}^{-1}]\hat{X}Q = 0\,.$$

Premultiplying here by $\hat{X}^{-1}$, the system is easily seen to reduce to

$$[(I - \alpha') - \varrho_0\hat{\gamma}\eta']Q = 0$$

which determines Q (but for a proportionality factor) exclusively in terms of α, γ, η, and ϱ_0.

This system can be written as

$$Q_i = \sum_j Q_j\alpha_{ji} + \varrho_0\gamma_i \sum_j Q_j\eta_{ji}$$

which is easily interpreted. $\Sigma_j Q_j\alpha_{ji}$ is the value received from other sectors per unit of output from sector i, and ϱ_0 is the "force of exploitation," which is applied to the value of the labour input per unit of output from sector i. The value of the labour input is determined by the value of the goods consumed by the workers.

The arguments above require that ϱ_0 be determined in terms of α, γ, and η by

$$\left|\frac{1}{\varrho_0}I - (I - \alpha')^{-1}\hat{\gamma}\eta'\right| = 0\,.$$

It is easily seen that this corresponds to the formula given by Morishima and Seton for the determination of ϱ_0.

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